Group insurance against common shocks [*]

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Abstract

We study insurance against common shocks in cooperatives and other productive groups of individuals. In those groups, and due to strategic interactions among group members, insurance decisions may be preferably taken at the group level rather than the individual level. We highlight two kinds of potential problems with individual insurance: the first one is a free-riding problem because due to strategic interactions, insurance decisions exert a positive externality on other group members; the second one is a coordination problem that occurs because it may be unprofitable for an individual to take insurance if the others in the group do not. Both types of problems can be resolved if insurance is offered at the group level.

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1 Introduction

Risk reduction remains a major challenge in increasing productivity and enhancing livelihoods among smallholders in developing countries. In the agricultural sector in particular, risk is often seen as a major ingredient of poverty traps. After the occurence of a negative income shock, agricultural households can reduce their children's school frequentation (Jacoby and Skoufias, 1997) or can be obliged to sell productive capital at low prices (Rosenzweig and Wolpin, 1993). In anticipation of the shocks, they can diversify their income by investing time in low productivity activities (Dercon and Krishnan, 1996), they can also fail to adopt high-yield but high-risk varieties (Eswaran and Kotwal, 1990).

The revenue of agricultural households in a community is subject to different kinds of shocks. A typology of shocks is possible according to the statistical properties of their distribution. The two polar types are idiosyncratic shocks and common shocks. Idiosyncratic shocks are independently distributed in the community. In a first approximation, health shocks are an example of idiosyncratic shocks. Common shocks affect everyone in the community. Price fluctuations and weather shocks are, again in a first approximation, examples of common shocks. Insurance against idiosyncratic shocks is possible via mutualization. In a celebrated empirical study, Townsend (1994) finds that village communities in three ICRISAT villages manage to achieve an outcome close to full insurance against idiosyncratic shocks via mutualization. Insurance against common shocks is much more difficult to achieve. It usually necessitates the intervention of third-parties like insurance companies and/or financial markets. For instance, insurance against price fluctuations is possible on derivative financial markets (Moschini and Lapan, 1995). Insurance against weather shocks requires the availability of suitable weather derivatives or the existence of re-insurance companies willing to diversify their portfolio. Of course the idiosyncratic or common nature of the shock depends on the perimeter of the community considered.

In this paper we are interested in the analysis of the demand for insurance against common shocks with a particular emphasis on insurance against weather shocks.

In the recent years there has been a growing interest in weather insurance for agricultural households in developing countries (see Barnett and Mahul, 2007, and World Bank, 2009). In particular, academic researchers and practitionners have argued that index-based policies can be used to insure poor rural households against weather shocks. Those policies make use of a meteorological index like the temperature or the cumulative rainfall in a given area, strongly correlated with the losses, and condition the payments to the insuree on the realized value of the index. Because the index measures a purely exogenous variable, offering these policies is not subject to moral hazard issues that usually plague insurance markets. Moreover, because information on past values of the index can be made public, there is no reason to suspect that adverse selection is a concern. Finally,

the recent availability of relatively cheap and reliable automatic meteorological stations has decreased the fixed cost associated to such policies. Index-based policies seem to be the right tool for insurance to reach the poor (Skees, Hartell and Yao, 2006). Since 2003, there has been a number of experimentations of weather index-based policies in Malawi (World Bank CRMG, 2009), Morocco (Stoppa and Hess, 2003), Peru and Vietnam (Skees, Hartell and Goes, 2007), India (Manuamorn, 2007) and many other developing countries. Yet, individual uptake for those policies has been disappointingly low (see for instance Giné and Yang, 2009, or Carter et al., 2010).

We shall argue that the demand for weather insurance, and other types of insurance against common shocks, could rise if policies are sold at the community level rather than the individual level.

Several explanations have been proposed to shed light on the low individual demand for weather insurance. It has been argued that insurance policies are complex products and that poorly educated agricultural households face difficulties to understand their interest. It has also been argued that weather risk typically involves small probabilities that people are likely to misevaluate. It also involves very long term cost-benefit analysis and present-biased households may underevaluate their potential losses. Others argued that agricultural households already adopted a wide array of risk coping strategies such as credit and savings and that weather insurance does not bring much to them (see Gollier, 1995). Finally, some authors have argued that interlinked transactions must be taken into account in order to understand the demand for weather insurance. In particular, it is plausible that, in village communities, formal weather insurance interacts with informal risk sharing. Informal risk sharing makes use of repeated interactions among group members. The fear of being excluded from the group and unable to benefit from the mutualization of risks in the future is used as a disciplining device. When people have the opportunity to obtain insurance from outside, they rise their reservation utility and informal risk sharing can be jeopardized. This argument originates in Attanasio and Rios-Rull (2000) and has been used by Clarke and Dercon (2009) to explain low individual demand for weather insurance.

This last explanation suggests that risk coping decisions of individuals in village communities are likely to exert externalities on other community members. And that offering insurance against common shocks at the community level can internalize those externalities. In this paper we propose a generic model to scrutinize this point. We highlight two distinct characteristics of insurance against common shocks in communities. The first characteristic is that insurance decisions taken by one individual may exert a positive externality on other community members. Therefore the demand for insurance may be plagued by a free-rider problem and it is plausible that the sum of the individual willingnesses to pay for insurance is lower than the group willingness to pay for insurance. The

second characteristic is that the value of insurance against common shocks can be positive or negative for an individual, depending on the insurance decisions of the other community members. The game played by community members when they choose whether or not to take insurance is, in some circumstances, a coordination game. And community members may fail to coordinate on the Pareto dominant outcome in which they all choose to take insurance.

Our model is built on the following specification for individual preferences : the utility of individual i in a group of N members depends on his own wealth and on the aggregate wealth of the group. Therefore, those individuals have social preferences. This is a rather natural hypothesis for village communities where interlinked transactions lead people to care about the wealth of others in the community. We propose the following rationale for this specification. If the group of individuals that we consider produces some local public good, then equilibrium utilities of individuals depend on those two variables. This remains true for several widely used individual preferences and under several decision rules for the provision of the local public good. We believe that this specification is particularly suitable to study the demand for weather insurance in agricultural cooperatives or other productive groups of individuals.

In such a setting, individual insurance decisions may exert a positive externality on others and create a free-riding problem. This may occur because the decision by one individual to take insurance involves a reduction in the risk associated to the aggregate wealth in the sense of second-order stochastic dominance. This will be valued by other group members provided the premium paid to get insured is not too high. As a consequence, the sum of the individual inverse demands for insurance, i.e. the sum of the individual risk premia, may be lower than what the group as a whole would be ready to pay, i.e. the group risk premium. Offering the insurance policy at the group level may rise demand.

When the two variables (i.e. own wealth and aggregate wealth) that enter the utility function of individuals are complements, a risk averse individual may prefer to stay uninsured if other group members do not take insurance. This occurs because individuals prefer to be rich when the group as a whole is rich and poor when the group as a whole is poor rather than poor when the group is rich and rich when the group is poor. Coordination of group members is necessary for uptake.

Two interesting papers, Clarke (2011a) and Clarke (2011b), study weather index-based and group-based policies from a different point of view. In Clarke (2011a), low demand for individual index-based insurance policies is explained by the low quality of such products. Because weather is not perfectly correlated with real shocks, there are some states of the world in which the agent pays for insurance, incurs losses and gets nothing from the policy. By contrast in our model insurance is perfect, i.e. is sold at the actuarially fair price and

completely smoothes revenue. Low demand is explained by externalities and free riding problems. In Clarke (2011b), group contracts for insurance policies are advocated on the basis of a costly state verification model. In such a model, group contracts are useful because they allow crowding in mutual insurance and reduce the overall verification cost. Our model is closer to the functioning of index-based policies and does not incorporate a verification of the losses.

The paper is organized as follows. In section 2 we present the model and some justifications. Examples of public good games are detailed to provide a rationale for the specification of individual preferences according to which both his own wealth and the aggregate wealth are taken into account. We relate such games to the functioning of small farmers' cooperatives in the developing world. We also justify the symmetry assumption that we make in sections 3 and 4 by studying the mutual insurance possibilities in the group. The core of the paper is sections 3 and 4. In section 3 we provide two illustrative examples. In the first one, we compare individual and group risk premia and show that the sum of individual premia can be lower than the group premium. In the second one, we show that insurance against common shocks can have a negative value. In section 4 we relate those illustrative examples to general properties of correlated stochastic variables and offer a more systematic analysis. Section 5 presents concluding remarks. All proofs are relaegated to the Appendix.

2   Social preferences and mutual insurance

2.1   Indirect utility

The community we consider is a group of N individuals. Each individual $i \in \{1, ..., N\}$ in the group is endowed with a wealth w_i. We denote by $W = \sum_{i=1}^{N} w_i$ the aggregate wealth in the group, and $W_{-i} = \sum_{j \neq i} w_j$ the sum of the wealths of individuals different from i. To take into account the fact that group members interact, we assume that each individual i has preferences given by the (indirect) VonNeumann-Morgenstern utility function

$$u_i(w_i, W) \tag{1}$$

With this specification, each individual cares about his own wealth but also about the aggregate wealth of the group. This kind of (indirect) social preferences is very plausible when the group considered is a cooperative or any other productive group. It is the result of having community members involved in several interlinked transactions with each others. We first provide different theoretical examples of settings where individuals have such preferences and then we explain how these can model the functioning of real-world producers cooperatives.

2.2 Public good provision

Consider a public good provision game among the N individuals. There are two goods in the economy : a private good c and a public good G. Each individual i can use his wealth w_i to buy a quantity c_i of the private good and contribute G_i to the public good. We normalize the price of both goods to be 1 so that the player's budget constraint is $w_i = c_i + G_i$. We also denote by $G = \sum_i G_i$ the total amount of the public good . The utility of individual i depends on his consumption of the private good and on the total amount of the public good that is available, i.e. it can be written

$$U_i(c_i, G).$$

In the sequel we will devote some attention to the following functional forms : the Cobb-Douglas function

$$U_i(c_i, G) = c_i^{\alpha_i} G^{\beta_i} \tag{2}$$

where the parameters α_i and β_i are strictly positive,

$$U_i(c_i, G) = \alpha_i \log(c_i) + \beta_i \log(G) \tag{3}$$

which is the log-linearized version of (2), and a constant elasticity of substitution function

$$U_i(c_i, G) = (\lambda_i c_i^{\alpha_i} + (1 - \lambda_i) G^{\alpha_i})^{\frac{1}{\beta_i}} \tag{4}$$

where $0 < \alpha_i \leq \beta_i < 1$.

Consider that contributions to the public good are mandatory, such as studied for instance in Epple and Romano (2003). They result from a collective choice rule adopted by the group. For simplicity let us assume that public good provision is financed through a proportional tax enforced at the group level. In that case, if we denote by T the tax rate, it is straightforward to establish that

$$U_i(c_i, G) = U_i\left((1 - T)w_i, T\sum_j w_j\right) = u_i(w_i, W).$$

With the preferences (2), (3) and (4), we obtain respectively up to an increasing and linear transformation :

$$u_i(w_i, W) = w_i^{\alpha_i} W^{\beta_i}, \tag{5}$$

$$u_i(w_i, W) = \log w_i + a_i \log W \text{ with } a_i > 0, \tag{6}$$

and

$$u_i(w_i, W) = (w_i^{\alpha_i} + b_i W^{\alpha_i})^{\frac{1}{\beta_i}}, \quad b_i > 0. \tag{7}$$

Moreover it can be shown (see Appendix A) that when individual preferences are given by (2) or (3), the preferred tax rate of individual i does not depend on the level of his

wealth w_i. This property ensures that any collective choice procedure based on individual preferences will select a tax rate that does not depend on the distribution of wealth in the group.

Consider now that individuals contribute non-cooperatively to the public good. We assume that the public good provision game is a strategic form game in which individuals decide simultaneously how much to contribute to the public good. When preferences are given by (2), (3) or (4) respectively, this game has a unique Nash equilibrium (see Bergstrom, Blume and Varian (1986) or Cornes and Hartley (2007)) which can be easily derived (see Appendix B). If in equilibrium every individual contributes a positive amount to the public good (which occurs when individuals are not too asymmetric), the equilibrium utilities are given respectively and up to an increasing and linear transformation by

$$u_i(w_i, W) = W^{\alpha_i + \beta_i}, \tag{8}$$

$$u_i(w_i, W) = \log(W), \tag{9}$$

and

$$u_i(W) = W^{\frac{\alpha_i}{i}}. \tag{10}$$

In these cases, the indirect utility depends only on the aggregate wealth in the community and not on the individual's own wealth.

2.3 Producers' cooperatives

In this subsection we explain why preferences given by equation (1) are relevant to model the situation of small producers that belong to a cooperative. We detail two relevant examples of the functioning of producers' cooperatives. These examples highlight the fact that public good provision is an important feature of those producers' cooperatives.

Cost-sharing cooperatives

A first category of cooperatives is designed to exploit economies of scale and share the burden of production fixed costs. Cooperative members share capital, administrative costs or marketing costs. These costs can be recurrent and are covered by the contributions of the cooperative members. Usually, the cooperative sets a rebate (a tax) on the unit price it pays to producers so that individuals contribute proportionally to their use of the capital and administrative or marketing tasks. To fix ideas, if each individual i produces q_i, he pays a share q_i/Q where $Q = \sum_j q_j$ of the fixed cost.

Suppose that individual i obtains a revenue w_i^1 in a first period that comes from the sale of his production to the cooperative. At this period he must decide how much of his revenue he consumes (c_i) and how much he invests for next year's production

(G_i). This investment captures decisions concerning the seeds, the fertilizer to be used, etc...For simplicity we assume that next year's production will be proportional to G_i and that the proportion is the same for all cooperative members. The revenue available to individual i in period 2 is therefore a function of his investment G_i but also of the aggregate investment G because this is what determines aggregate production or how the fixed cost will be shared in period 2. As a consequence, the decision problem faced by the cooperative members in period 1 is similar to the public good decision problem studied in the preceding subsection. They have to decide how to split their first period revenue between consumption of a private good and contribution to a public good. And equilibrium utilities can be written, from period 1 perspective, as in equation (1).

Collective asset cooperatives

A second category of cooperatives is designed to manage a collectively owned asset such as a financial contract or a sales contract. In these situations, the purpose of the cooperative can be to give access to the market and by-passing intermediaries. This is possible because the cooperative can contract on volumes that an individual producer is unable to guarantee. The purpose can also be to give access to formal credit. Again this is possible because the cooperative can provide some collateral that an individual producer could not provide. In these situation, the collective asset can be seen as a public good for cooperative members. Its management necessitates time and money to be fully profitable and cooperative members must contribute to it. Here, the link with the public good decision problem studied in the preceding subsection is straightforward. Collectively owned assets benefit everyone in the cooperative and generate free-riding problems. Whether the contribution to those public goods is voluntary or mandatory depends on the degree of institionalization of the collective decision processes in the cooperative. Mandatory contributions are more likely in cooperatives that rely on more formal transactions. For an array of contribution processes, equilibrium utilities of cooperative members can be written as in equation (1).

2.4 Mutual insurance

In this subsection we elaborate on the indirect utilities described in equation (1) to study the demand for insurance by individuals in the group. As a first step, we study what kind of insurance the group itself can provide to its members. Taking into account the possibility of such a mutual insurance mechanism will prepare the analysis conducted in sections 3 and 4. This will lead us to isolate the common component of the shock that impacts individual wealths and will provide a rationale for the symmetric setting we study in the next sections.

In order to introduce risk in the environment, we assume that the initial wealth profile $w = (w_1, ..., w_N)$ is a stochastic variable that takes values in $[\underline{w}, \bar{w}]^N$ and is distributed according to the joint density g. We denote by E_g the associated expectation operator. The wealth of individuals is therefore subject to shocks that can be idiosyncratic and/or common.

Risk-sharing inside the group, i.e. mutual insurance, can be used to provide insurance against idiosyncratic shocks. Mutual insurance consists in redistributing wealth among individuals without changing the aggregate wealth of the group. It will be valued by all individuals in the group provided their indirect utility exhibits risk aversion with respect to own wealth and individuals are not too different concerning their risk exposure, i.e. the shocks they face are not too asymmetric. We formalize this with the two following assumptions:

ASSUMPTION 1 : For each i, the indirect utility function $u_i(w_i, W)$ is increasing and strictly concave in the first argument.

ASSUMPTION 2 : For each i, the conditional expected value of w_i satisfies

$$E_g(w_i | \sum_{j=1}^{N} w_j = W) = \frac{W}{N}$$

.

Under ASSUMPTION 2 , given that the aggregate wealth is W, the expected wealth of agent i is W/N : a symmetry requirement.

Proposition 1 *Under* ASSUMPTIONS 1 *and 2, each individual in the group values from an ex ante perspective a mutual insurance agreement that results in an equal sharing of the aggregate wealth, i.e. each individual receiving W/N in all states of nature.*

Under ASSUMPTIONS 1 and 2, group members are likely to agree on an ex ante mutual risk sharing agreement. This agreement requires some form of commitment by group members since ex post, once the shock has occurred, some group members will want to renege on risk sharing. Such a commitment may be achieved via explicit incentives, i.e. an explicit binding contract, or via implicit incentives coming from repeated interactions among group members. It is beyond the scope of the present study to analyze under which conditions mutual risk-sharing is self-enforcing. We refer the interested reader to Ligon (2009) for an up-to-date treatment of that subject.

In the situation described in Proposition 1 the only remaining source of wealth variation will be common risk, i.e. variation in aggregate wealth. The group is unable to provide insurance to its members against this common risk. Such an insurance can only be provided by an external agent such as an insurance company.

Notice that when indirect utility is given by equations (8), (9) or (10), mutual insurance is unnecessary. In particular, when indirect utility depends only on aggregate wealth because it comes from a public good voluntary contribution game, individuals are already insured against idiosyncratic shocks by the public good contribution game. This is a direct consequence of the well-known fact that private provision of public good is independent of the distribution of income (Warr, 1983). When public good contribution is mandatory, however, the individual's wealth w_i directly influences his utility and mutual insurance is valuable.

3 Free-riding and coordination: illustrative examples

3.1 Free-riding

When preferences of individuals depend on their own wealth and on the aggregate wealth in the group it is likely that insurance decisions exert externalities. Indeed, when one individual takes insurance, this decision modifies the distribution of his own wealth and, by a composition effect, modifies also the distribution of the aggregate wealth. Therefore insurance of one individual may change the welfare of all individuals in the group. In what follows we provide an example where these externalities are positive externalities in the sense that the group is ready to pay more for insurance than the sum of what each individual is ready to pay.

We consider a group of N identical individuals whose indirect utility function is given as before by $u(w_i, W)$. We assume that the individual wealths w_i are subject to the same common shock, i.e. for all i, $w_i = w$ where w is a positive and non-degenerate random variable distributed according to g. The notation E_g (resp. $\hat{w}$) is used for the expectation operator (resp. the expected value of w).

To model the insurance decisions, we assume that an insurance company proposes to fully insure the risk, i.e. to replace the individual wealth w_i by its expected value $\hat{w}$, for a positive premium.[1]

ASSUMPTION 3 : The indirect utility function $u(w_i, W)$ is increasing in w_i, strictly increasing in W and concave.

Suppose that insurance is offered to the group and that its price is shared equally among group members. Let us denote c the per member price of insurance. Individuals

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[1]Full insurance is an extreme assumption that we adopt here in order to facilitate the interpretation of the results. The free-riding effect that we want to highlight in this Section would still hold with partial insurance but would be more difficult to isolate.

in the group will buy insurance whenever

$$E_g u(w, Nw) \geq u(\hat{w} - c, N(\hat{w} - c)).$$

We now define c^g the group risk premium, i.e. the highest price group members are ready to pay when insurance is offered at the group level. This risk premium is given by

$$E_g u(w, Nw) = u(\hat{w} - c^g, N(\hat{w} - c^g)) \qquad (11)$$

It exists and is positive and unique under ASSUMPTION 3.

Suppose now that insurance is offered to the individuals. Each individual i is offered insurance at a price c_i. Each individual i will buy insurance whenever

$$E_g u \left(w, w + (N - 1)\hat{w} - \sum_{j \neq i} c_j \right) \geq u \left(\hat{w} - c_i, N\hat{w} - \sum_{j \neq i} c_j - c_i \right)$$

Lemma 1 *Under ASSUMPTION 3, the maximal amount individuals are ready to pay for insurance is the same for all individuals in the group. It is given by c^i which solves*

$$E_g u \left(w, w + (N - 1)(\hat{w} - c^i) \right) = u(\hat{w} - c^i, N(\hat{w} - c^i)) \qquad (12)$$

When individuals are symmetric, the equilibrium of the insurance market is necessarily symmetric even when the insurance monopolist can prefectly discriminate: the risk premium is necessarily identical for all individuals.

We now provide an example in which $c^g > c^i$. We focus on the case where the individual indirect utility functions are given by equation (9) and are identical for all individuals in the group so that they are given by:

$$u_i(w_i, W) = \log(W). \qquad (13)$$

We first determine the risk premium associated to the lottery w, i.e. the maximal amount c that an agent is ready to pay to replace the lottery w by a constant wealth $\hat{w}$.

Suppose first, that insurance is offered at the group level and that the premium Nc is shared equally among agents. In that case, agent i is ready to pay an amount up to c^g with

$$E_g \log(w) = \log(\hat{w} - c^g) \qquad (14)$$

Suppose now that insurance is offered at the individual level. We focus on the risk premium that agent i is ready to pay, when the other agents take full insurance and pay a premium that results in a constant aggregate level of wealth $W_{-i} > 0$. Now, if agent i does not take insurance he gets

$$E_g \log(w + W_{-i}),$$

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while if he takes insurance for a premium c he gets

$$\log(\hat{w} - c + W_{-i}).$$

Let us denote c^i the individual risk premium for that agent.

Proposition 2 *Suppose that individual utility functions are given by equation (13), for each individual i, $c^i < c^g$.*

Here, when an agent decides to buy insurance coverage, he exerts a positive externality on the welfare of others.[2] When insurance is offered at the individual level, nobody internalizes those externalities and the equilibrium (potentially) results in underprovision, i.e. in our setting, $c^i < c^g$. The discrepancy between the individual and the group risk premia is non-negligible as the next numerical example shows.

EXAMPLE : To evaluate the difference between c^i and c^g, let us consider the following numerical application. The utility of individuals is $u(W) = \log W$, the individual wealth w takes value in $\{1, 2\}$ with equal probability $1/2$. The group risk premium c^g solves

$$\frac{1}{2}\log 2 = \log\left(\frac{3}{2} - c^g\right)$$

which gives

$$c^g = \frac{3}{2} - \sqrt{2} \approx 0,0858.$$

The individual risk premium c^i solves

$$\frac{1}{2}\left(\log\left(1 + (N-1)\frac{3}{2} - x\right) + \log\left(2 + (N-1)\frac{3}{2} - x\right)\right) = \log\left(N\frac{3}{2} - x\right)$$

For $N = 3$, straightforward computations give

$$c^i \approx 0,0283,$$

which implies that, in a group of three individuals, the sum of the individual premia is three times less than the group premium. □

To solve the free-riding problem, it is possible to offer the insurance at the group level. When group members have to chose between insurance for everybody in the group and insurance for nobody in the group they will be ready to pay the higher premium c^g. In order to extract this higher premium, it is important that individuals in the group are not offered the opportunity to buy individual insurance instead.

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[2]The positive externality is rigorously established in Section 4 for the setting we consider here. We prove there that, when indirect utilities are given by equation (13), actuarially fair insurance taken by one individual increases the welfare of others.

3.2 Coordination

In the preceding subsection we analyzed the individual incentives to take insurance under the hypothesis that the other group members purchase insurance. Whatever individual i decided, i.e. to take or not to take insurance, it was assumed that the other group members took insurance. As it will become clear in this section, fixing the decisions of the other group members was necessary because the indivudal incentives depend on the insurance decisions of the others. In fact, the externalities we highlighted there in the preceding subsection that purchasing or not insurance is a strategic decision. To gain a better understanding of this issue, it is useful to recast the insurance decision problem in terms of a strategic form game in which each group member must decide simultaneously to take or not to take insurance. In this game, the payoffs are the ex ante expected utilities (computed before the realization of the shock on the wealth distribution). We show below that this game can be a coordination game with multiple equilibria. In particular, we provide an example where insurance has a negative value : even if insurance is costless, nobody in the group wants to buy insurance if he (correctly) anticipates that the others in the group will not buy insurance. It illustrates the fact that the demand for insurance against common shocks can be plagued with multiple equilibria with either all agents or none being insured.

In this subsection it is assumed that insurance is perfect and sold at the actuarially fair price. This extreme hypothesis is considered in order to give the best chances to the insurance policy. With imperfect or costly insurance, the negative value result would only be more likely to hold.

Suppose that indirect utilities are given by equation (5) and are identical across individuals, i.e. for all i :

$$u_i(w_i, W) = w_i^\alpha W^\beta, \quad \alpha \gtrless 0, \ \beta > 0, \ \alpha + \beta < 1 \tag{15}$$

Suppose that agents can obtain full insurance for free, i.e. they can choose to exchange their stochastic wealth w for a certain wealth equal to the expected value $\hat{w}$. As mentionned above, we are interested in the strategic form game in which individuals simultaneously choose to take insurance or not to take insurance. The payoffs in this strategic form game are as follows. If k other individuals choose to take insurance, individual i gets

$$E_g u_i(\hat{w}, (k+1)\hat{w} + (N - k - 1)w) = \hat{w}^\alpha E_g((k+1)\hat{w} + (N - k - 1)w)^\beta$$

if he takes insurance and

$$E_g u_i(w, k\hat{w} + (N - k)w) = E_g w^\alpha (k\hat{w} + (N - k)w)^\beta$$

if he does not.

If all agents except agent i take the insurance, it is in the interest of agent i to take the insurance as well because in this case his utility is given by $w^\alpha(w + (N-1)\hat{w})^\beta$ which is concave with respect to w. Therefore the insurance game in which the agents simultaneously choose to take or not the free insurance always has an equilibrium in which all agents take the insurance. But it may not be the only equilibrium of that game.

To see this suppose that no other individual takes the insurance. If individual i does not take the insurance his payoff is

$$E_g w^\alpha (Nw)^\beta,$$

while if he takes the insurance, it is

$$\hat{w}^\alpha E_g(\hat{w} + (N-1)w)^\beta.$$

Proposition 3 *Suppose indirect utilities are given by equation (15) with $\alpha = \beta = 1/3$ and w is distributed on $\{0, \bar{w}\}$ with probabilities $\{p, 1-p\} = \{1/2, 1/2\}$. For $N \geq 7$, there is an equilibrium of the insurance game in which nobody takes the insurance.*

In that case, the insurance game possesses two equilibria : one with full insurance, i.e. insurance taken by all agents, the other with no insurance, i.e. insurance taken by no agent. Of course, the full insurance equilibrium Pareto dominates the no insurance equilibrium, nevertheless there is a priori no guarantee that agents will manage to coordinate on the full insurance equilibrium.[3] Group insurance can solve the coordination problem because it would let the group choose between the two equilibrium outcomes : full insurance or no insurance. There would be a unanimous agreement on the full insurance outcome.

4   Free-riding and coordination: more general results

In this section we go beyond the illustrative examples of the preceding section and scrutinize the conditions under which free-riding and coordination problems appear.

When individuals in a group have indirect social preferences as it is the case in our model, their welfare depends not only on the distribution of their own wealth but on the distribution of the wealth profile in the group. To study the behavior of invidual i it is not sufficient to scrutinize the properties of the real-valued stochastic variable w_i that describes his own wealth. It is necessary to scrutinize the properties of the stochastic

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[3]The game theory literature has repeatedly pushed forward the fact that in coordination games there is a priori no reason to focus exclusively on the Pareto dominant equilibrium, see for instance Harsanyi and Selten (1988), Carlsson and van Damme (1993).

variable $w = (w_1, w_2, ..., w_N)$ that takes value in $\mathbb{R}^N$ and describes the wealth profile in the group. Therefore we start this section by presenting some useful properties of multivariate distributions. In the following, X and Y are two stochastic variables that take values in $\mathbb{R}^N$

Definition 1 *The stochastic variable Y is a mean-preserving spread of the stochastic variable X when the equivalent statements (i) or (ii) hold*

(i) for all continuous and concave functions f, $E(f(Y)) \leq E(f(X))$;

(ii) Y has the same distribution as $\hat{Y}$ such that $(X, \hat{Y})$ is a martingale, i.e. $E(\hat{Y}|X) = X$.

The equivalence between the two statements is established by the Blackwell-Sherman-Stein theorem (see chapter 7 in [26], for instance). It follows that (ii) is a suitable definition of increasing risk because all risk averse decision makers, those whose preferences are represented by a concave function, prefer X to Y, according to (i). When the stochastic variables take values in $\mathbb{R}$, the following simple characterization is due to Rothschild and Stiglitz [25].

Definition 2 *When the stochastic variables X and Y take values in $\mathbb{R}$, Y is a mean-preserving spread of X if and only if (i) and (ii) hold*

(i) the two stochastic variables have the same mean, $E(Y) = E(X)$;

(ii) for all $x \in \mathbb{R}$,

$$\int_{-\infty}^{x} F_Y(v)dv \quad \int_{-\infty}^{x} F_X(v)dv,$$

where F_Y (resp. F_X) denotes the cumulative distribution of Y (resp. X).

In what follows, the wealth of individual i is given by a stochastic variable w_i. We denote by g_i the marginal distribution of w_i, E_{g_i} the expectation operator with respect to that distribution and $\hat{w}_i$ the mean value or expectation of w_i. In this setting, what we call insurance is the possibility for an individual to replace his own stochastic wealth w_i by its mean value $\hat{w}_i$.

Lemma 2 *Suppose that the individual wealths w_i are independently distributed. The stochastic variable $(w_1, ..., w_{j-1}, w_j, w_{j+1}...w_N)$ is a mean-preserving spread of the stochastic variable $(w_1, ..., w_{j-1}, \hat{w}_j, w_{j+1}, ..., w_N)$.*

When shocks on wealth are idiosyncratic, i.e. the random variables w_i, $i = \{1, ..., N\}$ are independently distributed, replacing the stochastic wealth of any individual j by its expected value $\hat{w}_j$ induces a reduction in risk. Therefore, any risk averse individual will

find insurance against idiosyncratic shocks profitable even if his preferences depend on the distribution of the whole wealth profile of the group.

When shocks on wealth are common then the random variables w_i, $i = \{1, ..., N\}$ are correlated. In this case the result no longer holds.

Lemma 3 *Suppose that the individual wealths are given by the same (non degenerate) stochastic variable w with mean $\hat{w}$. The stochastic variable $(w, ..., w)$ is not a mean-preserving spread of the stochastic variable $(w, ..., \hat{w}, ..., w)$.*

The fact that one individual insures himself against his own wealth variations does not imply a reduction in risk concerning the distribution of the whole wealth profile. As a consequence, when the utility of an individual depends not only on his own wealth but on the whole wealth profile, and even if this utility function is concave because the individual is risk-averse, insurance may be unvaluable.

Lemma 4 *Suppose that the individual wealths are given by the same (non degenerate) stochastic variable w with mean $\hat{w}$. The stochastic variable $(\hat{w}, ..., \hat{w}, w, \hat{w}, ..., w)$ is a mean-preserving spread of the (degenerate) stochastic variable $(\hat{w}, ..., \hat{w}, ..., \hat{w})$.*

Therefore, taking insurance has a positive value for a risk-averse individual if all the others take insurance. The problem is one of coordination because the value of being insured may depend on the decision of others.

To go a little further, we can also scrutinize the behavior of the aggregate wealth. We denote by k the number of group members that take insurance, the aggregate wealth is a real-valued stochastic variable

$$W_k = k\hat{w} + (N \quad k)w.$$

Lemma 5 *For $0 \le k < N$, the stochastic variable W_k is a mean-preserving spread of the stochastic variable W_{k+1}.*

When one individual takes insurance, it decreases the risk associated with the distribution of the aggregate wealth. If individuals care about the aggregate wealth of the group, in addition ot their own wealth, then it is clear that insurance decisions exert a positive externality. A risk-averse individual is better off when someone in the group takes insurance.

If indirect utilities depend only on the aggregate wealth of the group members, i.e. for instance are given by equation (8), (9) or (10), then insurance cannot have a negative value. We summarize this discussion and the consequences of lemmata 2 to 5 in the following Proposition.

Proposition 4 *Suppose the utility of individuals is given by concave functions $u_i(w_i, W)$,*

- *If $u_i(w_i, W) = u_i(w_i)$, i.e. preferences depend only on own wealth, then insurance is always positively valued by individuals ;*

- *If individual wealths are independent stochastic variables, i.e. shocks are purely idiosyncratic, then insurance is always positively valued by individuals;*

- *If individual wealths are given by the same stochastic variable, i.e. shocks are purely common, and individual preferences depend on the aggregate wealth, then insurance decisions exert positive externalities.*

- *If individual wealths are given by the same stochastic variable, i.e. shocks are purely common, and individual preferences depend only on the aggregate wealth, then insurance is always positively valued by individuals.*

To understand why insurance can have a negative value as in the example of Proposition 3, we must realize that individual insurance decisions have two effects. The first effect is a reduction of the risk associated to the variation in own wealth. This is certainly positively valued by risk avers individuals. The second effect is a modification of the joint distribution of (w_i, W). In particular, if shocks are highly correlated among individuals and individual i is the only one taking insurance, this may result in a lower correlation between the two variables w_i and W. In some circumstances, individuals prefer to have the two variables that enter their utility function highly correlated. This second effect can therefore be negatively valued and can also dominate the first effect. This was what happened for Proposition 3.

As it is now clearer, complementarities between the individual's wealth and the aggregate wealth of the other members of the group are key to explain the negative value of insurance at the individual level. Because of those complementarities, it is preferable for individual i that his own wealth is subject to the same shocks as the wealth of other group members, rather than being insured against these shocks. Beyond the example provided in the preceding section, we now present sufficient conditions on the indirect utility functions that guarantee that insurance against common shocks can have a negative value.

ASSUMPTION 4 : For each i, the indirect utility function $u_i(w_i, W)$ is increasing in the second argument, differentiable and such that for all w_i,

$$\lim_{W \to +\infty} \frac{\partial u_i}{\partial w_i}(w_i, W) = +\infty.$$

The last part of ASSUMPTION 3 is not equivalent to the hypothesis of constant sign of the cross-partial derivative $\frac{\partial^2 u_i}{\partial w_i \partial W}$, it neither implies or is implied by single-crossing. But

it is linked to it. It is another way to capture some elements of complementarity between the two variables. ASSUMPTION 4 is satisfied by the preferences given in equation (15).

Proposition 5 *Suppose the indirect utility functions of individuals satisfy* ASSUMPTIONS *1 and 4, then insurance against a common shock can have a negative value for all individuals.*

When indirect utilities are given by equation (7), ASSUMPTION 4 is satisfied. Therefore when individuals have indirect utilities that come from a constant elasticity of substitution function, insurance against common shocks can have a negative value.

5   Final remarks

The simple model we developped in this paper has shed light on two important phenomena that may arise when insurance against common shocks is offered to groups of interlinked individuals. First, in that context, insurance decisions may exert a positive externality and generate a free-riding problem if insurance policies are offered directly to the individuals. Second, contrary to what occurs with independent shocks, insurance against a common shock may have a negative value: if an individual anticipates that the others in the group will not take insurance, he may find insurance for himself unprofitable.

These two phenomena suggest that the demand for insurance against common shocks, for instance the demand for index-based weather insurance, may increase if policies are sold to intermediaries such as producers cooperatives.

Practitionners in the weather insurance sector are aware of the potential interest of dealing directly with cooperatives. As E. Meherette, Nyala Insurance S.C.'s deputy CEO explains:

> "Nyala has found that farmers' unions serve as effective delivery channels for the weather insurance products. By working with cooperative unions, Nyala insures all farmers who belong to the cooperative under the same contract. The cooperative is responsible for both paying the premium and distributing potential payouts to each insured farmer, reducing transaction costs for Nyala" (Meherette, 2009).

From the point of view of insurance companies, group policies certainly decrease the transaction costs. They also contribute to the scaling up necessary to recover fixed costs. Beyond these offer side advantages, we showed in this paper that group policies may also increase the demand for insurance.

In the microfinance sector, group contracts have already been pushed forward for credit. Group loans were suspected to explain the success of micro-credit institutions. Theoretical arguments have been proposed to explain their superiority over individual loans in terms of overcoming adverse selection and moral hazard (liability) problems (see Armendariz de Aghion and Morduch, 2005, for a synthesis of the different arguments).[4] The case for group insurance, as developed in this paper, relies on largely distinct arguments. In particular, group insurance must be targeted at groups that already exist and share a common interest. What occurs after the insurance contract is signed is not changed by the fact that it is a group contract. What is changed is mainly what occurs in interlinked transactions.

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A Mandatory contribution to the public good

Consider the case where preferences are given by equation (3). With a tax rate T, the utility of agent i is

$$\alpha_i \log((1 - T)\omega_i) + \beta_i \log\left(T \sum_{j=1}^{N} \omega_j\right)$$

Those functions are concave in T and the most preferred tax rate of agent i is

$$\bar{T}_i = \frac{\beta_i}{\alpha_i + \beta_i},$$

which does not depend on the wealth of individual i but only on his preferences parameters α_i and β_i. Suppose that the group's choice is driven by the median voter's preferences. Agents are ranked according to their $\frac{\beta_i}{\alpha_i + \beta_i}$, with

$$\frac{\beta_1}{\alpha_1 + \beta_1} \leq \frac{\beta_2}{\alpha_2 + \beta_2} \leq \cdots \leq \frac{\beta_N}{\alpha_N + \beta_N}.$$

The median voter si determined by the parameters α and β exclusively and does not change with the distribution of wealth in the population. This has the important implication that the vote on the tax rate can take place before or after the occurence of the shocks on wealth : this would not change the result of the vote. Let us denote m the median voter. The equilibrium condition is :

$$G^{eq} = \frac{\beta_m}{\alpha_m + \beta_m} \left(\sum_{i=1}^{N} \omega_i\right)$$

The equilibrium utility of agent i is therefore :

$$\alpha_i \log\left(\left(\frac{\alpha_m}{\alpha_m + \beta_m}\right)\right) + \beta_i \log\left(\left(\frac{\beta_m}{\alpha_m + \beta_m}\right)\right) + \alpha_i \log(\omega_i) + \beta_i \log\left(\sum_{j=1}^{N} \omega_j\right)$$

When preferences are given by equation (2), they are obtained by an increasing transformation of (3) and are therefore identical to the ones given by (3) as long as there is no uncertainty. The proof then follows. □

B Voluntary contribution to the public good

We derive the unique equilibirum of the voluntary contribution game by using the technique mentionned by Cornes and Hartley (2007). Consider the case where individual preferences are given by equation (3). The marginal rate of substitution of individual i is given by

$$MRS_i(c_i, G) = \frac{\alpha_i}{\beta_i} \frac{G}{c_i}.$$

For an aggregate level of public good provision equal to G, individual i maximizes its payoff if and only if his contribution G_i satisfies

$$G_i = r_i(G) = \max\{0, w_i \quad \frac{\alpha_i}{\beta_i}G\},$$

because either his contribution equals zero and the marginal rate of substitution is less than 1 (corner solution) or the marginal rate of substitution is equal to 1 (interior solution). By summing over i we obtain :

$$\sum_{i=1}^{N} r_i(G) = \sum_{i=1}^{N} \max\{0, w_i \quad \frac{\alpha_i}{\beta_i}G\}.$$

Let us denote

$$I(G) = \{i \in N : w_i \quad \frac{\alpha_i}{\beta_i}G > 0\}.$$

Equilibrium conditions are therefore

$$G^* = \frac{\sum_{i \in I(G^*)} w_i}{1 + \sum_{i \in I(G^*)} \frac{\alpha_i}{\beta_i}},$$

$$G_i^* = \max\{0; w_i \quad \frac{\alpha_i}{\beta_i}G^*\},$$

$$i \in I(G^*) \quad \text{if and only if} \quad w_i \quad \frac{\alpha_i}{\beta_i}G^* > 0.$$

When the players are not too asymmetric, we will have $I(G^*) = N$ i.e. all the players contribute a strictly positive amount to the public good. In that particular case, equilibrium conditions give:

$$G^* = \frac{\sum_{i=1}^{N} w_i}{1 + \sum_{i=1}^{N} \frac{\alpha_i}{\beta_i}},$$

$$c_i^* = w_i \quad G_i^* = \frac{\alpha_i}{\beta_i}\frac{\sum_{i=1}^{N} w_i}{1 + \sum_{i=1}^{N} \frac{\alpha_i}{\beta_i}}.$$

So that the equilibrium utility of a particular player i is :

$$\alpha_i \log\left(\frac{\alpha_i}{\beta_i}\right)\left(+ (\alpha_i + \beta_i)\log \quad \frac{1}{1 + \sum_{i=1}^{N} \frac{\alpha_i}{\beta_i}}\right) + (\alpha_i + \beta_i)\log \quad \sum_{i=1}^{N} \omega_i \right)($$

Here, the indirect utilities depend only on the aggregate wealth of the group.

Other specifications for the preferences of the individuals lead to indirect utility functions with W as the unique argument.

Suppose the utility of individuals, instead of being given by equation (3), is given by (2). These utility functions are increasing and concave transformations of the utilities given in equation (3). Therefore they represent the same preferences in riskless

environments. The equilibrium of the voluntary contribution game is unchanged. It is straightforward to show that the corresponding indirect utilities are now, up to an affine transformation :

$$u_i(w_i, W) = W^{\alpha_i + \beta_i}$$

Suppose now that the utility of individuals is given by a constant elasticity of substitution function such as (4. In that case, the marginal rate of substitution of agent i is given by

$$MRS_i(c_i, G) = \frac{i}{1 \quad i} \left(\frac{G}{c_i} \right)^{1 - \alpha_i}.$$

In an interior equilibrium of the voluntary contribution game, there is a linear relation between c_i and G with

$$G = \left(\frac{1 \quad i}{i} \right)^{\frac{1}{1 - \alpha_i}} c_i.$$

The same argument as developped above establishes that, in an interior equilibrium, the public good quantity G is proportional to the aggregate wealth W with

$$G = W \quad 1 + \sum_{i=1}^{N} \left(\left(\frac{i}{i} \right)^{\frac{1}{1 - \alpha_i}} \right)^{-1}.$$

Therefore, in an interior equilibrium, the indirect utility function of individuals is of the form

$$u_i(W) = W^{\frac{\alpha_i}{i}}.$$

C   Proofs

C.1   Proof of Proposition 1

In order to make use of ASSUMPTION 2, it is useful to decompose the expected utility as

$$E_g u_i(w_i, W) = E_g[E_g(u_i(w_i, W)|W)]$$

where $E_g(\cdot|W)$ denotes the conditional expectation operator. Under ASSUMPTION 1 we know that

$$E_g(u_i(w_i, W)|W) \leq u_i(E_g(w_i|W), W)$$

which, under ASSSUMPTION 2, induces

$$E_g u_i(w_i, W) \leq E_g u_i \left(\frac{W}{N}, W \right) ($$

As a consequence, from an ex ante perspective, each individual values a mutual insurance agreement.

C.2   Proof of Lemma 1

We prove by contradiction that individuals cannot differ in the maximal amount they are ready to pay for insurance, given that all the others pay their maximal amount and get insurance. Let us denote c_j^i the maximal amount individual j is ready to pay and assume that there exists k and l such that $c_k^i > c_l^i$. we know that

$$E_g u\left(w, w + (N-1)\hat{w} - \sum_{j \neq k} c_j^i\right) = u\left(\hat{w} - c_k^i, N\hat{w} - \sum_j c_j^i\right)($$

The fact that u is increasing in w_i ensures that

$$E_g u\left(w, w + (N-1)\hat{w} - \sum_{j \neq k} c_j^i\right) \leq u\left(\hat{w} - c_l^i, N\hat{w} - \sum_j c_j^i\right),$$

or

$$E_g u\left(w, w + (N-1)\hat{w} - \sum_{j \neq k} c_j^i\right) \leq E_g u\left(w, w + (N-1)\hat{w} - \sum_{j \neq l} c_j^i\right).$$

When u is strictly increasing in its second argument, this last equation is in contradiction with $c_k^i > c_l^i$. Therefore, the individual risk premium is necessarily the same for all agents and solves equation (12).

C.3   Proof of Proposition 2

From the definition of the individual risk premium c^i, and by denoting $W_{-i} = (N-1)\hat{w} - \sum_{j \neq i} c^i$, we know that

$$E_g \log(w + W_{-i}) = \log(\hat{w} - c^i + W_{-i}).$$

In words, c^i is the risk premium an individual with utility given by $\log(\cdot + W_{-i})$ would be ready to pay. Now we use the fact that the log function exhibits decreasing abolute risk aversion, so that an individual with preferences given by $\log(\cdot + W_{-i})$ is (strictly) less risk averse than an individual with utility given by $\log(\cdot)$. A standard result in risk theory (see Pratt (1964), or Gollier (2001) for a synthetic presentation) tells us that for any non-degenerate lottery, the risk premium of the latter individual is larger than the risk premium of the former. According to equation (14), c^g is the risk premium of the latter individual and we deduce that $c^i < c^g$.

C.4   Proof of Proposition 3

We have to compare the payoff of individual i without insurance

$$(1-p)N^\beta \bar{w}^{\alpha+\beta},$$

to the payoff with insurance

$$(1-p)^\alpha \bar{w}^\alpha (p((1-p)\bar{w})^\beta + (1-p)((1-p)\bar{w} + (N-1)\bar{w})^\beta.$$

After simple manipulations, we obtain that individual i prefers no insurance whenever

$$(1-p)N^\beta - (1-p)^{1+\alpha}(N-p)^\beta - p(1-p)^{\alpha+\beta} > 0.$$

Now take, as in the Proposition, $p = 1-p = 1/2$, $\alpha = \beta = 1/3$ and $N \geq 7$. It is immediate to verify that the inequality holds.

C.5   Proof of Lemma 2

It is straightforward to verify statement (ii) in Definition 1 with $Y = \hat{Y} = (w_1, ..., w_N)$ and $X = (w_1, ..., \hat{w}_j, ..., w_N)$.

C.6   Proof of Lemma 3

Consider two stochastic variables X and $\hat{Y}$ such that $X = (w, ..., \hat{w}, ..., w)$ and $E(\hat{Y}|X) = X$. If $\hat{Y}$ puts positive weights only on diagonal elements of the form $(y, y, ..., y)$, then $E(\hat{Y}|X)$ can only be a diagonal element. This is in contradiction with the equality $E(\hat{Y}|X) = X$ and the fact that X does not put positive weights only on diagonal elements because w is non degenerate (i.e. $w = \hat{w}$ is not always true). Therefore $\hat{Y}$ necessarily puts positive weights on elements outside the diagonal and cannot possess the same distribution as $Y = (w, ..., w)$.

C.7   Proof of Lemma 4

It is straightforward to verify statement (ii) in Definition 1 with $Y = \hat{Y} = (\hat{w}, ..., \hat{w}, w, \hat{w}, ..., \hat{w}))$ and $X = (\hat{w}, ..., \hat{w})$.

C.8   Proof of Lemma 5

The aggregate wealth W_k is less than a given threshold x whenever

$$(N-k)w + k\hat{w} \leq x,$$

or equivalently

$$w \leq \hat{w} + \frac{x - N\hat{w}}{N - k}.$$

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The right-hand side of the above inequality is increasing (resp. decreasing) in k when x is above $N\hat{w}$ (resp. below $N\hat{w}$). Therefore, the cumulative density F_{W_k} of W_k is increasing with k (resp. decreasing with k) above $N\hat{w}$ (resp. below $N\hat{w}$). As a consequence,

$$\int_{-\infty}^{x} \left(F_{W_k}(y) - F_{W_{k+1}}(y) \right) dy \le 0, \forall x \in \mathbb{R},$$

and W_k is a mean-preserving spread of W_{k+1}.

C.9 Proof of Proposition 5

We assume that the shock on individual wealths is common and given by the random variable w which takes value in $\{0, \bar{w}\}$, with $\bar{w} > 0$. The distribution of w is such that $w = 0$ with probability $1/2$ and $w = \bar{w}$ with probability $1/2$. Consider individual i and suppose the others do not take insurance. His expected payoff if he does not take insurance is

$$\frac{1}{2} u_i(\bar{w}, N\bar{w}) + \frac{1}{2} u_i(0, 0),$$

while if he takes insurance he gets

$$\frac{1}{2} u_i \left(\frac{\bar{w}}{2}, \frac{\bar{w}}{2} + (N-1)\bar{w} \right) + \frac{1}{2} u_i \left(\frac{\bar{w}}{2}, \frac{\bar{w}}{2} \right).$$

The agent strictly prefers not to take insurance whenever

$$u_i(\bar{w}, N\bar{w}) - u_i \left(\frac{\bar{w}}{2}, (N - \frac{1}{2})\bar{w} \right) > u_i \left(\frac{\bar{w}}{2}, \frac{\bar{w}}{2} \right) - u_i(0, 0).$$

Because the function u_i is increasing in its second argument and differentiable, we know that

$$u_i(\bar{w}, N\bar{w}) - u_i \left(\frac{\bar{w}}{2}, (N - \frac{1}{2})\bar{w} \right) \ge u_i(\bar{w}, N\bar{w}) - u_i \left(\frac{\bar{w}}{2}, N\bar{w} \right) = \int_{\frac{\bar{w}}{2}}^{\bar{w}} \frac{\partial u_i}{\partial w_i}(x, N\bar{w}) dx.$$

Because u_i is concave in w_i we obtain

$$u_i(\bar{w}, N\bar{w}) - u_i \left(\frac{\bar{w}}{2}, (N - \frac{1}{2})\bar{w} \right) \ge \frac{\bar{w}}{2} \frac{\partial u_i}{\partial w_i}(\bar{w}, N\bar{w}).$$

Under ASSUMPTION 4, the right-hand side of the last equation goes to $+\infty$ as N goes to $+\infty$. Suppose now that starting from an initial group of size N_0 with possible heterogenous individuals, we replicate this economy by creating k avatars of each initial individual-type. As k goes to $+\infty$, the size of the replicated group, $N = kN_0$, goes to infinity while keeping the number of individual-type fixed. Therefore it is possible to find k sufficiently high such that for all individual i in the group

$$u_i(\bar{w}, N\bar{w}) - u_i \left(\frac{\bar{w}}{2}, (N - \frac{1}{2})\bar{w} \right) > u_i \left(\frac{\bar{w}}{2}, \frac{\bar{w}}{2} \right) + u_i(0, 0).$$