Structural Change with Input-Output Linkage

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Abstract

This research examines the effect of intermediate input outsourcing on structural change. First we document a new fact: conditional on the traditional price and income effect, structural change happens from relatively intermediate input demandable sector to relatively suppliable sector. Then we build up a simple multi-sector partial equilibrium model to explain the mechanism and intuition. we find that sector with growing intermediate input demand would rely less on producing value added; sector with growing intermediate input supply would requires more primary input to produce. Primary input finally shift out from demandable sector to suppliable sector. we extend the reduced form model to a general equilibrium model. We endogenize the input-output linkage and price in function of outsourcing cost and TFP parameters. we model the inter-sectoral trade of intermediate input as analogous to international trade of final output in Ricardian trade framework following Eaton and Kortum (2002). We allow nonhomothetic CES preference in sectoral consumption. We calibrate the model to data for 35 major economies over 1995-2007. The quantitative analysis suggests that the intermediate input supply effect is the dominant effect on structural change, which is more than twice as large as the traditional price effect and income effect. The intermediate input demand effect is not as large as supply effect, because the intermediate input demand intensity is relatively stable over time.

Keywords: Input-Output Linkage, Structural Change, Outsourcing, Ricardian Trade, Nonhomothetic CES Preference.

JEL classification: O14, O41, O47, F11.

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1. Introduction

The world economy experiences substantial structural change in the last a few decades. In many developed economies including US, UK, Japan and Canada, the value added share increases in service sector and decreases in manufacturing sector since at least 1970. The similar structural change happens to emerging economies, such as China, India, Brazil and South Africa after 1990. This structural change fact has been well documented by many literature. At the same time, more and more manufacturing firms outsource their business service activities to specialized service companies. These service activities contain not only high skill services, such as professional service, accounting service, legal service and consulting service; but also low skill service, such as cleaning, security, janitorial service and logistic service. In particular as documented by Berlingieri (2013), Since 1947, professional and business services increased by 9.2 percentage points in US, which accounting for roughly 40 percent of the total growth in the entire service sector. More importantly, 91 percent of the business service output are sold to firms as intermediate input, which is much higher than 53 percent for the whole economy.

In this research, we examine the effect of intermediate input outsourcing on structural change. The key structure term focused by this research is value added share at sector level. But we also talk about consumption share and employment share in the content. Following Helpman (2006), outsourcing is defined as the acquisition of an intermediate input or service from an unaffiliated supplier. At firm level, it refers to the central ”make-or-buy” decision which has been discussed by classic transaction cost theory by Coase (1937) and Williamson (1985). In this research firms faces the decision of whether produce (make) intermediate input in-house or outsource (buy) intermediate input from other firms. Since different firm has very different outsourcing decision, it results in heterogeneous sector and industry level intermediate input demand and supply intensity. My research question therefore is if we directly take the sectoral level input-output linkage into account, does this heterogeneous variation of linkage implies structural change?

We find that intermediate input outsourcing does affect structural change. Conditional on the traditional price and income effect, structural change happens from relatively intermediate input demandable sector to relatively suppliable sector. The intuition is that for the sector with growing demand of intermediate input, it relies more on intermediate input rather than primary input to produce. Then the primary input such as labour and capital would shift out from this sector. On the other hand, for the sector with growing supply of intermediate input, it requires

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1For recent review of structural change, see Herrendorf, Rogerson and Valentinyi (2013a).
2It could be one digit sector, such as manufacturing. It could also be two or more digit level, such as miscellaneous professional, scientific, and technical services. In this research we mainly focus on one digit sector. But we also show the fact in more disaggregated level.
more primary input to produce gross output in order to meet the growing outsourcing demand. Therefore labour and capital shift from relatively demandable sector to relatively suppliable sector. Our quantitative result suggests that the intermediate input supply effect is the dominant effect on structural change, which is more than twice as large as the traditional price effect and income effect. The intermediate input demand effect is not as large as supply effect, because the intermediate input demand intensity is relatively stable over time.

To document this relationship, we need to formally define the key relevant attribute of intermediate input linkage. We define the sectoral intermediate input supply and demand multiplier as the column and row vector sum of Leontief inverse matrix respectively. Correspondingly this supply and demand multiplier measures the interconnection of a sector to all other sectors through the supply and demand mechanism of intermediate input. In the literature of shock transmission, the supply multiplier is called "forward linkage" or "influence vector". We use the intermediate input supply and demand multiplier as two sufficient statistics to study the relationship.

We find that the time series pattern of intermediate input supply multiplier matches with the corresponding sectoral value added share. Figure 1 presents one example of this fact. The top panel shows the US value added share in two sectors: Food and beverage and tobacco products (311FT) and Administrative and support services (561). Indeed over the long run time period 1963-2015, the value added share decreases substantially from more than 3 percent to about 1 percent in a three digit manufacturing sector (311FT); whereas the value added share in a three digit service sector (561) at 2015 is almost 3 times as large as in 1963. This different pattern of value added share represents one example of the overall US structural change from manufacturing sector to service sector in the last five decades. Moreover, the lower panel of figure 1 shows the corresponding time series pattern of intermediate input supply multipliers for these two sectors. The supply multiplier of the manufacturing industry decreases substantially during this period while the supply multiplier of the service industry shifts up dramatically. More importantly, the long run pattern of value added share is roughly consistent with the corresponding sectoral intermediate input supply multiplier. Figure 1 suggests a strong positive correlation between the sectoral intermediate input supply multiplier and value added share.

To explain the positive correlation between intermediate input supply multiplier and value added share, we first use a multi-sector partial equilibrium model following Jones (2011). In this model, every sector uses intermediate inputs to produce gross output. On the other hand, we allow every sector to supply intermediate inputs to other sectors. By solving the profit maxi-

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3See Acemoglu, Carvalho, Ozdaglar and Tahbaz-Salehi (2012)
mization problem in competitive equilibrium, the intuition is obvious to see. Leontief inverse matrix plays a central role. The element value of Leontief inverse matrix actually is understood as elasticity of sectoral gross output to TFP. For example, if any sector increases TFP by one percent, the corresponding row vector of Leontief inverse would tell us the percentage increase in all other sectors’ gross output. Not surprisingly the aggregate value added would be increased by a weighted average of the the corresponding row vector of Leontief inverse. Therefore the sum of row vector of Leontief inerse which is defined as intermediate input supply multiplier, is a sufficient statistic to determine the elasticity of aggregate value added (GDP) to sectoral TFP. More importantly, this elasticity is also the sector size in terms of gross output relative to GDP. Since sectoral value added is a share of sectoral gross output, it implies that the supply multiplier positively determines the sectoral value added share.

On top of the qualitative relationship, we further examine the quantitative importance of the intermediate input supply effect on structural change. In order to independently compare the
supply effect with the traditional price effect and income effect\textsuperscript{4}, we extend the partial equilibrium model to a general equilibrium model. In consumer’s side, we use non-homothetic CES preference (Comin et al. 2015) to allow constant elasticity of substitution between sectoral consumption; and simultaneously heterogeneous income elasticity for different sectoral consumption. This framework enables us to independently isolate the price effect from income effect. As discussed in Comin, Lashkari and Mestieri (2015), this non-homothetic assumption is also consistent with the empirical fact.

In producer’s side, we use Ricardian trade framework to endogenize the intermediate input supply effect and price effect. In this research, we take the input-output linkage as the inter-sectoral trade of intermediate input. More importantly, this domestic inter-sectoral trade of intermediate input is analogous to the classic international trade of final output. Specifically we follow Boehm (2015) by applying the Eaton and Kortum (2002) trade framework to sector pair trade of intermediate input. Similar to Eaton and Kortum (2002), the model could be solved tractably though we have detail information at both sector and firm level. In equilibrium, both the input-output linkage and price are endogenous on TFP parameter and inter-sectoral trade cost.

Following the quantitative model, we apply the model result to data. The main database we use in this research is World Input-Output Database (WIOD). This database not only provides long run input-output tables for 40 major economies, but also contains important economic accounting information on its Social Economic Accounts (SEA) dataset, such as price deflator, value added, fixed capital and employment. For the sectoral PPP price deflator data, we use the Sector and Industry Relative Price Database (SIRP) which is developed by Inklaar and Timmer (2014). The final panel data contains 35 countries, 4 sectors over 1995-2007\textsuperscript{5}. In addition, for longer time period input-output table data, I use the Input-Output Accounts Data from Bureau of Economic Analysis; and National Accounts Main Aggregate Database from UN.

The quantitative result suggests that the intermediate input supply effect is very significant and is the most dominant effect on structural change, compared with price effect and income effect. The model implies a log-linear relationship between structural change and the individual effect. Therefore we directly use OLS to estimate the coefficients of each individual effect. We find that both supply effect and price effect are significant at 1 percent level. More importantly, the intermediate input supply coefficient has a larger size than price effect. It suggests more response of structural change to intermediate input supply than price. In contrast, income effects

\textsuperscript{4}The seminar paper for price effect on structural change is Ngai and Pissarides (2007), for income effect see Kongsamut, Rebelo and Xie (2001). See literature review for model detail.

\textsuperscript{5}I remove Taiwan due to no price data; remove Bulgaria due to impossible price spike; remove Russia, India and Indonesia because the model result cannot be calibrated to their data.
are not significant for most studies, and are relatively trivial. In addition, the model also implies negative intermediate input demand effect. The demand effect is based on sectoral intermediate input demand share. The regression result suggests that the demand effect is negative and significant at 1 percent level.

In order to structurally estimate the four individual effect, I conduct counterfactual study by following the structural model. First, I calibrate the model parameter with the panel data in hand. Except for the Frecet distribution variation parameter, all other parameters are exactly calibrated to match with the input-output data and price index data. Actually the low value of Frecet distribution variation parameter is a common value used by literature. Second, we use the calibrated model to simulate the counterfactual study. We focus on two counterfactual studies. In the first one, we assume that the sector pair trade cost stays at the initial year level for all sample countries. In the second one, we assume that the sectoral TFP stays at the initial year level for all sample countries. With these two counterfactual setup, we simulate the endogenous input-output linkage and price, and also simulate the model generated value added share. Then we re-estimate the four individual four effects on structural change. The counterfactual study result confirms the regression result. That is, the intermediate input supply effect is the dominant effect; price effect is significant, but have less magnitude than supply effect; intermediate input demand effect is significant, but overall the effect is small due to relatively smaller variation in intermediate input demand intensity; income effects mostly are insignificant and trivial.

This research makes three contributions. First, we find a new fact about the relationship between structural change and inter-sectoral trade of intermediate input. Structural change happens from relatively more intermediate input demandable sector to more intermediate input suppliable sector. This is the first time this fact being explicitly and clearly documented. Second, this new fact can be explained intuitively by a simple partial equilibrium model. We can fully understand the intuition from the equilibrium result. More importantly, the partial equilibrium result holds in the general equilibrium quantitative model. Third, this research construct a very tractable quantitative model by merging the Ricardian trade idea with recently developed structural change framework. This research is the first research as far as we know clearly identify the dominant role of intermediate input supply effect, compared with the classic price effect and income effect.

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6The same low value of variation parameter is recently used by Eaton, Kortum, Neiman and Romalis (2016). In addition, Ruhl et al. (2008) argues that low frequency data should implies low estimate of elasticity of substitution in trade literature.

7This finding is consistent with literature. Many papers including Jones (2011) and Duarte and Restuccia (2017) argues for relatively stable intermediate input demand over time.
Related Literature  This research relates to three strands of literature. First, this research contributes to the study of structural change. The seminar paper of price effect on structural change is Ngai and Pissarides (2007), which generalize the idea of Baumol (1967). Ngai and Pissarides (2007) presents a multiple sector model with homothetic preferences. Their model implies that labour moves from low price (high productive) sector to high price (low productive) sector if consumption goods are complement. On the other hand, Kongsamut, Rebelo and Xie (2001) argues that different income elasticity between sectors can explain structural change and balanced growth at the same time. In their model agriculture, manufacturing and service have different income elasticity. Given income increases, the income elastic sector (service) will grow faster and therefore attract more employment than income inelastic sector (agriculture). This research discusses the importance of price effect and income effect as well. The result tends to support the significance of price effect; but not for income effect.

In terms of the relationship between input-output linkage and structural change, the most related papers are Berlingieri (2013) and Sposi (2015). Both of them argue that on top of final demand, intermediate demand is also important to affect structural change. In Berlingieri (2013), the sharp rise in the forward linkage of professional and business services in US implies that this industry has increasing influence on the rest of economy. Given more outsourcing from manufacturing to service, the employment share increases in service and decreases in manufacturing. Compare to Berlingieri (2013), this research generalise the finding to 35 economies, three digit industries. By studing value added share rather than employment share, this research avoids the need of equivalent capital share assumption. More importantly, while Berlingieri (2013) examines the supply effect in the similar reduced form way as we did, the reduced form model cannot structurally estimate the supply effect and cannot independently compare the supply effect with price effect and income effect. This is exactly what we focus in this research. Our research explicitely argue the dominance of intermediate input supply effect, by independently comparing the four individual effects. Sposi (2015) argues that the heterogeneous intensity of intermediate input demand contributes to the different economic structure across countries. In contrast, this research focuses on the time series pattern of sectoral intermediate input supply effect on structural change.

Second, in terms of the structural model, this research borrows the modeling framework from Ricardian trade literature. We model the domestic inter-sectoral trade of intermediate input in a similar way to the international trade of final output as in Eaton and Kortum (2002). This

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8Baumol (1967) presents an early example to explain the implication of different sectoral productivity growth on labour reallocation. According to Baumol (1967), to ensure balanced growth due to government intervention or due to inelastic demand and elastic income, labour has to move away from high productive sector to low productive sector. This
application idea follows Boehm (2015). We apply the Eaton and Kortum (2002) trade framework to every possible sector pair. Different to Eaton and Kortum (2002) which assumes that every firm has a choice to buy final output from many possible countries, every firm now has a binary choice between outsourcing and production in house. In equilibrium, the intensity of input-output linkage and price are endogenous on iceberg type of trade cost and TFP parameter. This helps us to solve the potential endogeneity problem in the empirical strategy, when we regress the structural change on intermediate input supply effect and price effect. The most important reason to use Eaton and Kortum (2002) framework is that, it provides a very tractable way to link the gravity equation in trade environment to intermediate input outsourcing decision within individual firms. Moreover, the individual firm level decision can finally being aggregated to sector level without losing tractability. Since this research aims for clear identification of the four individual effects, this tractability attribute is crucial.

Third, the endogeneity of intermediate input outsourcing on trade cost and technology follows a large literature. There are at least three related costs. The contract cost theory (Coase 1937; Williamson 1985; Tadelis 2002; Boehm 2015) argues that higher contract enforcement cost discourages firm outsource decision. Recently Dustmann, Fitzenberger, Schönberg and Spitz-Oener (2014) and Goldschmidt and Schmieder (2017) argue that the gradually less protection in low skill service occupant from labour union contributes to the rise of intermediate input outsourcing in Germany manufacturing firms since 1990. In addition, bad institution is likely to hold up comparative advantage (Nunn 2007), which is further likely to hold up intermediate input outsourcing (Boehm 2015). For the effect of technology change on intermediate input outsourcing, Bartel, Lach and Sicherman (2012) shows that technological change increases the likelihood of outsourcing, because of the benefit from leading-edge technologies. Furthermore, Abramovsky and Griffith (2006) suggests that the development of information and communication technology (ICT) increases the adaptability and compatibility of many service, which facilitates the tradability of services across firms.

Outline Section 2 presents the motivating facts. Section 3 uses a multi-sector partial equilibrium model to show the mechanism, intuition and a simple example of the mechanism. The general equilibrium model and the equilibrium result are illustrated in section 4. We present the quantitative analysis in section 5. Section 6 concludes.
2. Facts

In this section, we present four related facts. First, for major economies, Value added (VA) share drops in manufacturing and rises in service since 1970. If any, this trend is more obvious in the last two decades. Second, VA share increases with intermediate input supply multiplier for US three digit industries during 1963-2015. Third, the second fact holds for 4 sectors, 38 major economies over 1995-2007. It holds even for real value added share. Last, relative price of manufacturing to service matches very well with relative nominal value added share; not very well with relative real value added share. We introduce the data first, then followed by detail of the four facts.

2.1 Data

The nominal value data are available in the WIOD. We obtain the sectoral nominal value added panel data from the SEA dataset. We obtain the sectoral nominal value of intermediate inputs from the world input-output tables (WIOT). Since we do not consider the international trade, we ignore the import and export data. Similar we also ignore the government consumption, tax/subsidy and transport margin data. For most major economies, these values are relatively small in determining the economic structure and should not affect the result systematically. The sectoral nominal gross output value equals the summation of value added and the relevant intermediate input values.

The SIRP database from Inklaar and Timmer (2014) is important to transfer the nominal values into real values. Specifically we use the time series price deflators to calculate the real value of value added, intermediate input and gross output at 2005 local price. The SEA dataset provide all the relevant value added price deflators and gross output deflators. In the model we assume no distortion between the gross output price and intermediate input price, therefore we use the gross output price delators to deflate the nominal value of intermediate input. Then we aggregate the 35 individual sectors into 4 sectors following Inklaar and Timmer (2014). Finally we use the PPP deflators from the SIRP database and the annual exchange rate from the WIOD to calculate the real gross output, real value added and real intermediate input in 2005 global reference prices.

The real primary inputs are calculated in a similar way. The SEA dataset provides sectoral real capital stock value in 1995 local price. We calculate the real capital stock in 2005 local price by using the capital price deflators in SEA. Then we use the capital PPP deflators from PWT 8.1 to calculate the real capital stock in 2005 global price. For the labour input, we use the total employment hours. Alternative choice could be number of employees. Many papers including
Fadinger, Ghiglino and Teteryatnikova (2017) argues for the choice of total employment hours. Because data on hours worked allows accounting for differences in working patterns, for instance, full-time and part-time workers.

The final panel data are available for 4 sectors and 38 countries over 1995-2007. The real capital data are not available in the year 2008 and 2009 for a few European countries in the WIOD 2013 database. In addition, since the SIRP database does not provide PPP price deflator data for Taiwan, we drop out Taiwan from the sample. In addition, we also use National Accounts Main Aggregate Database (NAMAD) from UN and US input-output table from Bureau of Economic Analysis (BEA). These two databases enable us to track value added and input-output linkage for longer time period.

2.2 Fact 1

The first fact is not new to this research. It has been documented by many structural change researches. We present this fact for 4 developed economies in figure 2: US, UK, Japan and Canada;
and for 4 emerging economies in figure 3: Brazil, India, South Africa and Turkey\textsuperscript{10}. These two figures present the time series pattern of value added share for manufacturing sector and service sector since 1970. The horizontal axis is year; and vertical axis is value added share in percentage.

Figure 2 and figure 3 suggest that in these sample countries, value added share decreases in manufacturing sector and increases in service sector since 1970. This fact is obvious for both developed economies and emerging economies. These economies not only have very different development process, but also very different culture, history, population size and geography. Actually we could find similar structural change fact in other major economies as well. It suggests that at least since 1970 the major economies experience significant structural change from manufacturing sector to service sector.

In addition, structural change is more obvious to see after 1990, in particular true for emerging economies. We can see that structural change tend to accelerate in UK and Canada after

\textsuperscript{10}The long run data is not available for China in NAMAD.
1990. The value added share of manufacturing sector drops more significantly than before; on the other hand value added share of service sector rises more significantly than before. For emerging economies, structural change is not obvious before 1990. Since 1990, we can see very significant structural change in all of the emerging economies in figure 3. This fact has two implications. First, Value added share is one of the important structure term which depicts the structural change fact in major economies\textsuperscript{11}. Second, we do not miss out the structural change fact by using the main panel data WIOD which contains data between 1995 and 2007.

2.3 Fact 2

Before we introduce the second fact, we formally define the intermediate input supply and demand multiplier. Suppose B is the input-output linkage matrix which we normally observe in

\textsuperscript{11}For value added share based structural change, see Herrendorf, Rogerson and Valentinyi (2013a) and Rodrik (2016)
input-output table data. The well-known Leontief inverse matrix $L$ is defined by

$$L = (I - B)^{-1}$$

Given Leontief inverse $L$, we define the sectoral intermediate input supply and demand multiplier as the following

$$\mu_{it}^s = \sum_{j=1}^{n} L_{ij}$$
$$\mu_{jt}^d = \sum_{i=1}^{n} L_{ij}$$

In Acemoglu, Carvalho, Ozdaglar and Tahbaz-Salehi (2012), they call the intermediate input supply multiplier as forward linkage, and call the vector of supply multipliers as influence vector. We show the positive relationship between value added share and intermediate input supply multiplier for US three-digit sector in the second fact. we show this fact for farm sector in figure 4;
for sector of motor vehicles, bodies and trailers, and parts in figure 5; and for sector of miscellaneous professional, scientific and technical services in figure 6. Basically each figure shows one example of agriculture sector, manufacturing sector and service sector respectively. In every figure, the top panel shows the annual pattern of value added share; the middle panel shows the annual pattern of intermediate input supply multiplier; the bottom panel shows the annual pattern of relative intermediate input supply multiplier to demand multiplier.

Figure 4 to figure 6 suggest a strong positive correlation between US sectoral value added share and the corresponding intermediate input supply multiplier during 1963-2015. For the agriculture and manufacturing sector, the decreasing value added share pattern matches with decreasing intermediate input supply multiplier; for the service sector, the increasing value added share pattern also matches with increasing intermediate input supply multiplier. In addition, these figures suggest that the pattern of supply multiplier is very consistent with the pattern of relative supply to demand multiplier. This finding implies that intermediate input demand multiplier is relatively stable, compared with supply multiplier. The stable intermediate input
demand multiplier is consistent with Jones (2011) and Duarte and Restuccia (2017), in which they find that US sectoral intermediate input demand intensity is stable over time. Furthermore, figure 5 suggests that the relative supply to demand multiplier matches with value added share better than supply multiplier only. This implies that demand multiplier is also important to determine value added share. This finding motivates the next fact.

2.4 Fact 3

In this section, we present the central fact of this research. That is, there is a significantly positive relationship between sectoral intermediate input supply multiplier and sectoral value added share. On the other hand, there is a significantly negative relationship between sectoral intermediate input demand multiplier and residual sectoral value added share. The residual here means the remaining value added share after removing the supply effect in a simple linear regression.

These two relationships are presented in figure 7 and figure 8 respectively. In both figures, there are four panels. Each panel shows the correlation for an 1 digit sector: other good sector; manufacturing; market service and non-market service. The definition and disaggregation of each sector are given in Inklaar and Timmer (2014). The correlation is based on panel data of 38
major economies during 1995-2007. The horizontal axis in figure 7 and figure 8 are intermediate input supply multiplier and demand multiplier respectively. The vertical axis is sectoral nominal value added share for both figures. The positive correlation between value added share and supply multiplier is consistent with fact 2. Fact 3 suggests that this positive correlation is a general finding, which is true not only for long run time series data, but also true across countries. More importantly, we can observe this positive correlation for all sectors.

In addition, the strong negative correlation between intermediate input demand multiplier and sectoral value added share suggests that demand multiplier also matters. This negative correlation is illustrated in figure 8. In this figure the sectoral value added share is residue value added share. Figure 8 suggests that after removing the intermediate input supply effect, sectoral value added share is negatively correlated with the corresponding intermediate input demand multiplier. Notice that the sectoral value added share itself is not significantly correlated to intermediate input demand multiplier. It implies that intermediate input supply multiplier has a first order important correlation with sectoral value added share; while intermediate input demand multiplier only has a second order important correlation.
2.5 Fact 4

The last fact documents the price effect on structural change. According to Ngai and Pissarides (2007), if manufacturing and service are complement for consumption, structural change happens from relatively low price sector to relatively high price sector. It implies that we should see a positive correlation between relative price and relative value added share of manufacturing to service. We present this correlation for four developed economies and four developing economies in figure 9 and figure 10 respectively. Actually the sampling countries and time period are exactly the same as in fact 1. In both figures, the horizontal axis is year; the vertical axis is the index of relative value added share and relative price of manufacturing to service. The index is normalized to be 1 in year 1970.

Figure 9 and figure 10 suggest that relative price tracts with relative value added share very well for most of countries and for most of time. In this time period, relative value added share of manufacturing to service drops particularly for developed countries. This is consistent with fact 1. At the same time we observe that the relative price of manufacturing to service also de-
creases. There is a strong positive correlation between relative price and relative value added share, which is consistent with the prediction in Ngai and Pissarides (2007). Later on in the model and empirical section, we will directly consider this price effect. We also compare the price effect with intermediate input supply effect, to see which effect is stronger.

The upshot of this section is that the correlation between value added share and intermediate input supply and demand is very strong, very robust to long run time series, panel data, 3 digit sectors. In the rest of this research we try to explain the correlation, and quantitatively examine and compare those effects on structural change.

3. Reduced Form Mechanism

This section explains the mechanism of intermediate input supply and demand effect on structural change. In order to clearly explain the mechanism and economic intuition, we build up a multi-sector partial equilibrium model following Jones (2011). Though this is a very simple reduced form model, it is able to explain the mechanism. I show the mechanism first, then I
illustrate the mechanism in a simple example.

### 3.1 Mechanism

Assume there are n sectors in an economy. The sectoral gross output is produced in a constant return to scale Cobb-Douglas function form.

\[
Q_i = A_{GOi} K_i^{1-\sigma_i} L_i^{1-\sigma_i} (1-\alpha_i) \prod_{j=1}^{n} X_{ij}^{\sigma_{ij}}
\]

Here \(\sigma_i = \sum_{j=1}^{n} \sigma_{ij}\); \(K_i\) and \(L_i\) stand for capital and labour demand of sector \(i\) respectively; \(X_{ij}\) represents the intermediate input produced from sector \(j\), and used by sector \(i\); \(A_{GOi}\) is gross output based total factor productivity in sector \(i\). The gross output is either consumed as final goods or supplied to other sectors as intermediate inputs.

\[
C_j + \sum_{i=1}^{n} X_{ij} = Q_j
\]

The aggregate final good (real GDP) is assumed to be Cobb-Douglas composite of each individual sectoral consumption, such that

\[
Y = \prod_{i=1}^{n} C_i^{\lambda_i}
\]

Here \(\lambda_i\) is consumer preference of consumption from sector \(i\) and \(\sum_{i=1}^{n} \lambda_i = 1\).

Assume all markets are competitive, and capital and labour are inelastically supplied. In equilibrium, the sectoral value added share and employment share are endogenous on input-output linkage in the following way:

\[
\eta_i = (1-\sigma_i)\gamma_i
\]

\[
l_i = \frac{L_i}{L} = \frac{(1-\sigma_i)(1-\alpha_i)\gamma_i}{\sum_{i=1}^{n}(1-\sigma_i)(1-\alpha_i)\gamma_i}
\]

Here \(\gamma\) is Domar weight: \(\gamma = (I - B)^{-1}\lambda\); \(\sigma, \alpha\) and \(\lambda\) are sectoral intermediate input share, capital share and consumption share respectively. If we assume the same capital share across sectors such that \(\alpha_i = \alpha_j\), value added share equals employment share which is given by

\[
\eta = l = (I - \Sigma)(I - B)^{-1}\lambda
\]

Here \(\Sigma\) is a diagonal matrix with \(\Sigma_{ii} = 1 - \sigma_i\).

The mechanism is understood according to equation (1). We can start with the Leontief inverse matrix. Suppose we observe the sectoral input-output linkage matrix \(B\). The element \(L_{ij}\)
suggests that if sector i increases TFP by 1 percent, it would finally increases the sectoral gross output j by $L_{ij}$ percent. This final effect summarises all the direct and indirect effect of sectoral TFP shock, through the input-output linkage. Moreover, if gross output of sector j increases by $L_{ij}$ percent, aggregate value added would increases by $\gamma_i$ percent. It implies that the Domar weight $\gamma_i$ is aggregate output elasticity to sectoral gross output based TFP in sector i. In a corresponding case of value added economy, the sectoral value added share $\eta_i$ is aggregate output elasticity to sectoral value added based TFP in sector i\(^{12}\). Equation (1) implies a positive log-linear relationship between these two elasticities.

The larger the intermediate input supply multiplier, the larger the corresponding sectoral Domar weight. The intermediate input supply multiplier is a vector sum of the corresponding row in Leontief inverse. The Domar weight is a weighted average of the corresponding row in Leontief inverse. The sectoral weight is sectoral consumption share. Theoretically both the variation of weight and intermediate input supply multiplier contribute to the variation of Domar weight. Conditional on consumption share, stronger intermediate input supply implies higher Domar weight. Higher Domar weight implies larger elasticity of aggregate output to sectoral gross output based TFP, which implies this sector has larger size in terms of gross output. According to equation (1), it would also suggests higher elasticity of aggregate output to sectoral value added based TFP, which equivalent to larger sector size in terms of value added. Conditional on consumption share, for any sector with larger intermediate input supply multiplier, this sector will grow larger. On the other hand, for any sector with larger intermediate input demand multiplier, it implies larger intermediate input demand intensity. According to equation (1), this implies smaller value added share. In conclusion, conditional on consumption share, structural change happens from relatively larger intermediate input demand multiplier to relatively larger intermediate input supply multiplier sector.

The economic intuition is consistent with literature. First, it is consistent with structural change literature with input-output linkage. Both Sposi (2015) and Berlingieri (2013) build up a similar reduced form structural change equation as in equation (1). But they do not summaries the intuition in a general way like this. Second, the intermediate input supply effect is consistent with the central effect of sectoral shock in Real Business Cycle literature. For instance, Acemoglu, Carvalho, Ozdaglar and Tahbaz-Salehi (2012) define the intermediate input supply multiplier as forward linkage. They argue that the larger value of forward linkage of this sector, the larger influence this sector has in transmission of shock. That is why the vector of intermediate input supply multiplier is called the influence vector in Acemoglu, Carvalho, Ozdaglar and Tahbaz-

\(^{12}\)See Hu (2017).
Salehi (2012). Here we do not address sectoral shock transmission given input-output linkage. We put forward a different argument about structural change. We argue that the larger influence of this sector, the more resource this sector attract from other sectors.

### 3.2 Example

In this section I illustrate the intermediate input supply and demand mechanism in a very simple example. Though the example is simple, it is capable of explaining the mechanism well.

Assume an economy has two sectors: sector 1 and sector 2. Each sector produces one sector specific gross output. The gross output is either supplied to consumer as final consumption, or is supplied to another sector as intermediate input. For instance, we can think about these two sectors as electronic product sector and accounting service sector. On one hand electronic manufacturer requires accounting service to produce; on the other hand accounting service company also requires electronic product in providing accounting service. Their product or service are consumed by consumer as well.

Table 1 presents the structural change in this example. In the benchmark panel, we show the gross output (Q), consumption (C), value added (VA), intermediate input supply (IIS) and intermediate input demand (IID) before the structural change. This table mimics a two sector input-output table. The first two columns show the demand side of production; and the first two rows show the supply side of production. On demand side, each sector produces 1 unit of intermediate input in house; outsources 1 unit of intermediate input from other sector; and produces 2 units of value added. On supply side, each sector supply 1 unit of intermediate input to itself; supply 1 unit of intermediate input to other sector; and supply 2 units of final goods to consumer. Both sectors produce 4 units of gross output in the benchmark case.

Suppose there is a chock in sourcing cost. After the sourcing cost shock, sector 2 demands 1 more unit of intermediate input from sector 1; simultaneously sector 1 is able to supply 1 more
unit of intermediate input to sector 2. For instance, the de-unionization in European manufacturing sector substantially reduce the outsourcing cost from service sector, which induces manufacturers to outsource some of their traditional service activities to service companies\(^\text{13}\). In this case, sector 2 now depends more on intermediate input, therefore do not need to produce as much value added as in the benchmark case. Sector 2 outsources the service activities to sector 1, and focus on the main manufacturing production which produces value added in 1 unit. In contrast, sector 1 now is responsible for supplying more intermediate input to sector 2. Sector 1 accepts the outsourcing activities from sector 2, therefore sector 1 now needs to produce more value added in order to meet the additional supply responsibility. Not surprisingly, labour and capital would shift out from sector 2 to sector 1, to meet the relative change of value added production between these two sectors.

In conclusion, the shock of outsourcing cost leads to change pattern of inter-sectoral trade of intermediate input, which finally results in structural change of primary input and value added. This example has two important implications. First, the mechanism in this example is consistent with the general mechanism in the former section. That is, since sector 2 becomes relatively more intermediate input demandable; and sector 1 becomes relatively more intermediate input suppliable, the structural change happens from sector 2 to sector 1 at the end. Second, this example also motivates the model set-up in the next section. That is, the shock of outsourcing cost could be an important factor to contribute to change in inter-sectoral trade of intermediate input. In the next section, we directly model the path from outsourcing cost to inter-sectoral trade of intermediate input.

4. Model

In this section we extend the reduced form model to a multi-sector general equilibrium model. Three reasons stand out for the need of structural model. First, the reduced form model is silent about many endogeneity concerns. For instance, we take the consumption share as given in reduced form model. However, consumption share itself is an important economic structure term in structural change literature. Both price effect (Ngai and Pissarides 2007) and income effect (Kongsamut et al. 2001) are likely to contribute to consumption share change. Therefore they tend to contribute to value added share based structural change as well. In the structural model, we directly address the endogeneity of consumption share following Comin, Lashkari and Mestieri (2015). On top of that, we also model the endogeneity of price and intermediate

\(^{13}\)see the argument and examples of de-unionization in Goldschmidt and Schmieder (2017) and Dustmann, Fitzenberger, Schönberg and Spitz-Oener (2014).
input linkage. The model strategy is mapping the domestic inter-sectoral trade of intermediate input to international trade of final output in a Ricardian trade framework. In this sense, we borrow the modeling framework heavily from Boehm (2015) and Eaton and Kortum (2002).

Second, this general equilibrium model allows us to compare the intermediate input supply and demand effect with the traditional price and income effect. In contrast, we cannot independently see the four individual effects in the reduced form model. Thanks to the nonhomothetical CES preference assumption which is a main contribution in Comin, Lashkari and Mestieri (2015), we are able to isolate the four individual effects on structural change. Third, the model result implies a clear and simple identification strategy for our quantitative analysis in the next section. Basically the model implies that the structural change variable is a log-linear function of the four individual effects. More importantly, both price and input-output linkage are endogenous on outsourcing cost and TFP parameters. Overall on one hand the model contains a great deal of detail of production and consumption decision at firm level, sector level and aggregate level; on the other hand the model is very tractable to guide our quantitative study.

4.1 Preference

Assume there is a representative consumer. The representative consumer consumes final goods and services from n sectors. In addition assume the representative consumer has constant relative risk aversion utility function, which is given by

$$U(C_t) = \sum_{t=0}^{\infty} \delta^t C_t^{1-\phi} \frac{1}{1 - \phi}$$

(2)

Here $\delta$ is the discount factor; $1/\phi$ is elasticity of intertemporal substitution of aggregate consumption $C_t$. Following Comin, Lashkari and Mestieri (2015), assume that the aggregate consumption $C_t$ has a nonhomothetic Constant Elasticity of Substitution (CES) sum of intratemporal sectoral consumption $C_{it}$. The specific aggregation equation is defined by an implicit function, which is

$$\sum_{i=1}^{n} \Omega_i^{\frac{1}{\epsilon}} C_t^{\frac{\epsilon-\epsilon_i}{\epsilon}} C_{it}^{\frac{\epsilon-1}{\epsilon}} = 1$$

(3)

Here $\epsilon$ is elasticity of substitution between sectoral consumption; $\epsilon_i$ measures the income elasticity of demand for consumption good in sector i. Therefore in this setup, we allow heterogeneous income elasticity across sectors\textsuperscript{14}. This nonhomothetic CES sum is the key assumption to

\textsuperscript{14}Homothetical CES is a special form when we assume constant unitary income elasticity. That is, if $\epsilon_i = 1$, $C_t = \left(\sum_{i=1}^{n} \Omega_i^{\frac{1}{\epsilon}} C_{it}^{\frac{\epsilon-1}{\epsilon}}\right)^{\frac{1}{\epsilon}}$. 

independently identify price effect and income effect on consumption, as specified in equation \( \text{(28)} \). In terms of structural change, this assumption is important to isolate the price effect from income effect as illustrated in equation \( \text{(27)} \) and equation \( \text{(32)} \).

Assume the representative consumer has an initial asset endowment of \( A_0 \). The standard budget constraint equation is

\[
\sum_{i=1}^{n} P_{it} C_{it} = w_t L_t + (1 + r_t)A_t - A_{t+1} + \Pi_t
\]

Here \( \Pi_t \) is the aggregate profit; \( w_t L_t \) is aggregate labour income. We assume that the representative consumer owns all of the individual firms. According to Herrendorf, Rogerson and Valentinyi (2013a), the consumer’s optimization problem could be decomposed by two separate problems. The intertemporal problem maximizes the aggregate utility which is defined in equation \( \text{(2)} \) by allocating \( C_t \) optimally over all time periods. The intratemporal problem is a static problem which maximizes aggregate consumption \( C_t \) at a given time period \( t \) by allocating \( C_{it} \) optimally. Since only sectoral allocation is useful for structural change study, the focus is on intratemporal problem in this research\(^{15}\). To be specific for the intratemporal problem, the representative consumer maximize \( C_t \) subject to aggregate consumption definition equation \( \text{(3)} \) and budget constraint equation \( \text{(4)} \).

### 4.2 Technology

Assume aggregate gross output has a nonhomothetic CES sum of sectoral gross output, which is given by

\[
\sum_{i=1}^{n} \Psi_{it}^{\frac{\xi_i}{\rho}} Q_{it}^{\frac{\xi_i-\rho}{\rho}} Q_{it}^{\frac{\rho-1}{\rho}} = 1
\]

Here \( Q_t \) is aggregate gross output, and \( Q_{it} \) is sectoral gross output. Here we assume a similar form of nonhomothetic CES sum as in consumer’s problem. There is only one difference, and it is worth to emphasis the difference. Here we allow time-variant sectoral weight, which plays an additional role to price and income in determining structural change. Not surprisingly intermediate input supply multiplier could be one of the proxy to represent the sectoral weight. Because both the empirical fact and the mechanism suggests the importance of intermediate input supply effect on structural change. For model generality here, we allow a broadly defined sectoral weight. Overall the symmetrical nonhomothetic CES assumption between production and consumption is necessary to isolate the intermediate input supply effect on structural change. In

\(^{15}\)For the discussion of intertemporal problem and the related balanced growth path, see Comin, Lashkari and Mestieri (2015) for more detail.
particular, the structural change equation (33) builds up on this assumption.

We have two different notions about how sectoral gross outputs are produced. In the first notion, sectoral value added and the corresponding composite of material together contribute to production of sectoral gross output. The value added itself comes from primary input, such as capital and labour. Based on the first notion, people normally assume that sectoral gross output is produced in a constant return to scale Cobb-Douglas function form

\[
Q_{it} = A_{GOit} K_{it}^{(1-\sigma_{it})\alpha} L_{it}^{(1-\sigma_{it})(1-\alpha)} M_{it}^{\sigma_{it}}
\]

(6)

The term \( K_{it} \) and \( L_{it} \) stand for capital and labour demand of sector \( i \) at time period \( t \) respectively. The term \( A_{GOit} \) is gross output based TFP of sector \( i \) at \( t \). The term \( M_{it} \) represents the composite of outsourcing intermediate inputs or material good from all potential sectors.

In the second notion, sectoral gross output is directly taken as composite of individual sectoral intermediate inputs from all sectors. Here we need to differentiate the source of intermediate input between from outsourcing and production in-house (insourcing). At sector level if a sector \( i \) directly buy intermediate input from sector \( j \), we define it as intermediate input outsourcing. For the same intermediate input which sector \( j \) is able to produce, but producer in sector \( i \) chooses to produce it by hiring labour and capital without buying from sector \( j \) we define this situation as intermediate input production in-house\(^{16}\). In the first notion, the intermediate input outsourcing is summarised by \( M_{it} \); and the intermediate input production in-house is summarised by value added terms: \( K_{it} \) and \( L_{it} \). In the second notion we skip this intermediary terms, and directly mapping outsourcing and in-house production of intermediate inputs into sectoral gross output \( Q_{it} \).

The upshot is that the two notions are essentially the same. Equation (6) under the first notion is equivalent to the reduced form production equation in the last section. Actually the reduced form model holds if we turn off the endogeneity process, and aggregate production in-house into value added. Therefore the structural model in this section only extends the reduced form model in the last section, we do not deviate from the reduced form model. The first notion is comprehensive at sector level, which is useful to depict sectoral level variables. In this model, the first notion is mainly used to depict sector level structural term in proposition 1. The second notion is specific at individual sectoral level intermediate input production, which could be further extended to study firm level decision of intermediate input outsourcing and production in-house. Therefore the second notion is useful for examining the endogenous response of

\(^{16}\)Here the producer decision is whether outsourcing or production in-house (Boehm 2015). In trade literature, the decision is whether offshoring (including FDI) or producing domestically (Bernard et al. 2017). In firm organization literature, the decision is make-or-buy (Williamson 1985).
intermediate input outsourcing decision at firm level.

Following the second notion we assume that sectoral gross output is a homothetic CES sum of individual sectoral intermediate inputs, specifically given by

$$Q_{it} = \left( \sum_{j=1}^{n} X_{ijt}^\theta \right)^{\frac{1}{\theta}}$$  \hspace{1cm} (7)

Here $X_{ijt}$ is the necessary sectoral intermediate input for sector $i$ which could be outsourced from sector $j$ at time period $t$, or could be produced in house by hiring relevant professional labour and capital by individual firms under sector $i$. Notice that without considering firm level intermediate input sourcing decision, $X_{ijt}$ is taken as total sectoral intermediate input outsourcing from sector $j$ to sector $i$, as in Jones (2011). Given firm level outsourcing decision which will be specified below, $X_{ijt}$ contains both in-house production and outsourcing of intermediate inputs. Assume there are continuum of differentiated intermediate input varieties $\omega \in [0, 1]$ being used by each sector. Each variety is produced by one individual firm. Assume the sector-to-sector intermediate input pair is a CES sum of individual intermediate input variety, such that

$$X_{ijt} = \left[ \int_0^1 X_{ijt}(\omega) \frac{\nu}{\nu - 1} d\omega \right]^\frac{\nu}{\nu - 1}$$  \hspace{1cm} (8)

Here $X_{ijt}(\omega)$ is intermediate input variety $\omega$ which could be outsourced from individual firm under sector $j$ and used by another individual firm under sector $i$, at time period $t$. For the firm level intermediate input variety sourcing decision, I borrow the Ricardian inter-sectoral trade idea developed by Boehm (2015) which follows the international trade framework in Eaton and Kortum (2002). For the production of intermediate input variety $X_{ijt}(\omega)$, firm can either choose to produce in house by hiring labour and capital; or choose to outsource from another firm, which belongs to sector $j$ and also being able to enforce the firm-to-firm outsourcing contract. Assume every individual firm produces a differentiated intermediate input variety and the market of intermediate input variety is monopolistic competitive. Suppose a firm choose to produce in house the corresponding output and price are given by

$$X_{ijt}^H(\omega) = a_{ijt}^H(\omega) k_{ijt}^\alpha(\omega) l_{ijt}^{1-\alpha}(\omega)$$  \hspace{1cm} (9)

$$P_{ijt}^H(\omega) = \frac{\nu}{\nu - 1} \tilde{r} a_{ijt}^H(\omega)$$  \hspace{1cm} (10)

Here $\tilde{r} = \left( \frac{\nu}{\alpha} \right)^\alpha \left( \frac{\nu}{\nu - 1} \right) \nu^{1-\alpha}$; $X_{ijt}^H(\omega)$ and $P_{ijt}^H(\omega)$ are in-house production and price of intermediate
input variety $\omega$; $k_{ijt}$ and $l_{ijt}$ are firm level capital and labour demand of producing variety $\omega$ of intermediate input pair $ij$ during time period $t$ respectively; $a^H_{ijt}(\omega)$ is firm level in-house productivity of producing variety $\omega$ of intermediate input pair $ij$ during time period $t$. Every firm is assumed to have the same mark-up which is $\frac{\nu}{\nu - 1}$. On the other hand if the intermediate input variety is outsourced from sector $j$, suppose there is always one firm in sector $j$ can directly use the sectoral gross output as input to produce. Assume there is a sector-to-sector distortion$^{17}$ $\tau_{ijt}$ if sector $i$ outsources intermediate input from sector $j$ in time period $t$. Therefore here we assume distortion is sector specific, not firm specific. One example of this distortion is intermediate input contract enforcement cost between sectors as suggested by Boehm (2015). Specifically the production function and the corresponding price equation in this case are given by

\begin{align}
X^X_{ijt}(\omega) & = a^X_{ijt}(\omega)Q_{ijt}(\omega) \\
P^X_{ijt}(\omega) & = \frac{\nu}{\nu - 1} a^X_{ijt}(\omega) \tau_{ijt} \frac{P_{jt}}{a^X_{ijt}(\omega)}
\end{align}

Here $X^X_{ijt}(\omega)$ and $P^X_{ijt}(\omega)$ are production and price of outsourcing intermediate input variety respectively; $a^X_{ijt}(\omega)$ is firm level outsourcing productivity of producing variety $\omega$ of intermediate input pair $ij$ during time period $t$. Similar to Eaton and Kortum (2002) which assumes country specific Fréchet distributed firm level productivity, in this model we assume that firm level productivity follows sectoral specific Fréchet distribution:

\begin{align}
Pr[a^H_{ijt} \leq a] & \equiv F_{it}(a) = e^{-T_{it}a^{-\zeta}} \\
Pr[a^X_{ijt} \leq a] & \equiv F_{jt}(a) = e^{-T_{jt}a^{-\zeta}}
\end{align}

Here $T_{it}$ and $T_{jt}$ are sector specific parameter of TFP scale. This assumption suggests that any sector with larger value added based sectoral TFP has larger firm level productivity draw on average. The assumption of same parameter of $\zeta$ across sectors suggests that the variance of firm level productivities are assumed to be essentially the same in all sectors. Assume every intermediate input variety is perfectly substitutable between in-house and outsourcing production. The price of intermediate input variety $\omega$ producer of sector $i$ actually pay is the minimum price between in-house and outsourcing prices:

\begin{align}
P^*_{ijt}(\omega) & = \min(P^H_{ijt}(\omega), P^X_{ijt}(\omega))
\end{align}

$^{17}$Cost and distortion are used interchangeably in this research.
The trade-off between in-house and outsourcing decision is that outsourcing firm may be more productive but outsourcing naturally bear with sourcing cost. If sourcing cost is too high, firm is likely to produce in house. Otherwise, firm may consider outsourcing.

4.3 Equilibrium

At every time period $t$, the equilibrium is characterized by consumer’s optimal allocation of sectoral consumption and producers’ optimal allocation of primary input and intermediate input. Producer’s intratemporal problem can be further divided into two levels: sector level and firm level. At sector level, assume there is a representative producer which maximizes sectoral profit. Assume individual producer maximizes the respective profit equations or minimizes the cost equations by optimally allocate primary input, overall material good, sectoral intermediate input and individual intermediate input variety, subject to the corresponding budget constraints or production constraints.

At every time period, all markets are clear in equilibrium. There are three strands of market clearing conditions. First, the supply of sectoral gross output equals the demand of sectoral gross output. The demand contains final consumption (final demand) and intermediate input outsourcing (intermediate demand) from all possible sectors. Specifically we have the following equation.

$$P_{jt}C_{jt} + \sum_{i=1}^{n} P_{ijt}X_{ijt} = P_{jt}Q_{jt}$$  \hspace{1cm} (16)

Second, in the first notion of production side assume that capital market and labour market are clear during every time period. Assume capital and labour are supplied inelastically. Then the market clear conditions are given by

$$\sum_{i=1}^{n} K_{it} = K_t$$  \hspace{1cm} (17)

$$\sum_{i=1}^{n} L_{it} = L_t$$  \hspace{1cm} (18)

Third, in the second notion of production side we define a series of market clearing condition by the following equations:

$$P_{ijt}X_{ijt} = \int_{0}^{1} P_{ijt}(\omega_X)X_{ijt}(\omega_X)d\omega_X$$  \hspace{1cm} (19)

$$P_{ijt}X_{ijt} = \int_{0}^{1} P_{ijt}(\omega_H)X_{ijt}(\omega_H)d\omega_H$$  \hspace{1cm} (20)
\[ P_{ijt}X_{ijt} = P_{ijt}^X X_{ijt} + P_{ijt}^H X_{ijt} = \int_0^1 P_{ijt}^\omega (\omega) X_{ijt}(\omega) d\omega \]  
(21)

\[ P_{it}Q_{it} = \sum_{j=1}^n P_{ijt}X_{ijt} \]  
(22)

Here \( \omega_X \) and \( \omega_H \) are those of variety which are produced by outsourcing and in-house production respectively. Actually the two notions of sectoral gross output production are connected by the following equations:

\[ \sum_{j=1}^n P_{ijt}^H X_{ijt} = wL_{it} + rK_{it} \]  
(23)

\[ \sum_{j=1}^n P_{ijt}^X X_{ijt} = P_{Mit}M_{it} \]  
(24)

Again these two notions of production are exactly the same since we have the following equivalence:

\[ P_{it}Q_{it} = \sum_{j=1}^n P_{ijt}X_{ijt} = \sum_{j=1}^n P_{ijt}^X X_{ijt} + \sum_{j=1}^n P_{ijt}^H X_{ijt} = P_{Mit}M_{it} + wL_{it} + rK_{it} = P_{it}Q_{it} \]

After solving the model (all proof of this and other propositions are shown in appendix A), the following proposition summarises the key result of structure terms:

**Proposition 1** (Equilibrium Solution of Structure Terms and Outputs): Assume there is a representative producer which maximize the sectoral gross profit. In competitive equilibrium with optimal allocation of intratemporal consumption and inputs, the key structure terms are solved as the following

\[ \eta_{it} = \frac{P_{it}Y_{it}}{P_{it}Y_t} = (1 - \sigma_{it})\gamma_{it} \]  
(25)

\[ \gamma_t = (I_n - B_t)^{-1}\lambda_t \]  
(26)

\[ \lambda_{it} = \frac{P_{it}C_{it}}{P_{it}C_t} = \Omega_i \left( \frac{P_{it}}{P_t} \right)^{1-\varepsilon} C_{t}^{\varepsilon i} \]  
(27)

Here \( \eta_{it} \), \( \gamma_{it} \) and \( \lambda_{it} \) are sectoral value added share, Domar weight and consumption share respectively. The term \( P_{Yit}Y_{it} \) and \( P_{Yit}Y_t \) are sectoral and aggregate nominal value added; \( B_t \) is the sectoral input-output linkage matrix; \( \lambda_t \) is the vector of consumption share; \( \gamma_t \) is the vector of Domar weight with \( \gamma_{it} = \frac{P_{it}Q_{it}}{P_{it}Y_{it}} \). In addition, the optimal sectoral consumption and gross output are solved as

\[ C_{it} = \Omega_i \left( \frac{P_{it}}{P_t} \right)^{-\varepsilon} C_{t}^{\varepsilon i} \]  
(28)
\[
Q_{it} = \Psi_{it}^\kappa \left( \frac{P_{it}}{P_t} \right)^{-\rho} Q_t^{\xi_i} 
\]

\[
P_t = \left( \sum_{i=1}^{n} \Omega_i P_{it}^{1-\varepsilon} C_t^{\epsilon_i-1} \right)^{\frac{1}{1-\varepsilon}} 
\]

\[
P_t = \left( \sum_{i=1}^{n} \Psi_{it}^\kappa P_{it}^{1-\rho} Q_t^{\xi_i-1} \right)^{\frac{1}{1-\rho}} 
\]

Proposition 1 suggests that the economic structure terms are consistent with that of reduced form. Basically equation (25) and equation (26) are equivalent to equation (1) in section 3. According to proposition 1, the consumption is endogenous on sector price and aggregate consumption. It implies that price effect and income effect contribute to consumption share based structural change, which is discussed in proposition 2. Not surprisingly the sectoral gross output has a similar function form to sectoral consumption. But the intensity on price and income are allowed to be different. The weight in sectoral gross output are time variant. The parameter \( \kappa \) denotes the intensity on weight, which is the key parameter to determine the intensity of intermediate input supply effect on structural change.

Based on proposition 1, we derive the formal structural change equation in the following proposition.

**Proposition 2** (Structural Change): The structural change equations of relative sectoral consumption share and value added share are derived as the following

\[
\log \frac{\lambda_{it}}{\lambda_{jt}} = \log \frac{\Omega_i}{\Omega_j} + (1 - \varepsilon) \log \frac{P_{it}}{P_{jt}} + (\varepsilon_i - \varepsilon_j) \log C_t 
\]

\[
\log \frac{\eta_{it}}{\eta_{jt}} = \log \frac{1 - \sigma_{it}}{1 - \sigma_{jt}} + \kappa \log \frac{\Psi_{it}}{\Psi_{jt}} + (1 - \rho) \log \frac{P_{it}}{P_{jt}} + (\xi_i - \xi_j) \log Q_t 
\]

Proposition 2 provides the benchmark equations to study structural change. Proposition 2 suggests that structural change comes from three different economic forces in an input-output linkage environment: price effect, income effect, and intermediate input outsourcing effect. The first two effects are well studied in previous structural change literature, such as price effect in Ngai and Pissarides (2007) and income effect in Kongsamut, Rebelo and Xie (2001). More recently these two effects are independently identified by Herrendorf, Rogerson and Valentiny (2013b), Boppart (2014) and Comin, Lashkari and Mestieri (2015).

However, none of them can identify the intermediate input outsourcing effect, due to ignorance of intermediate input linkage at sectoral and firm level. If we only study the consumption share based structural change, the ignorance probably is fine following equation (32). But if we
want to know the forces behind value added based structural change, the ignorance will miss out the outsourcing effect. Specifically equation (33) suggests that any sector with faster growing domestic demand of intermediate input outsourcing, would reduce value added share. The opposite is true for intermediate input supply.

An recent empirical evidence which is mentioned by Goldschmidt and Schmieder (2017), Dustmann, Fitzenberger, Schonberg and Spitz-Oener (2014) and Berlingieri (2013) tends to support this. Since the mid-1990s there has been a dramatic increase in the outsourcing in business service sectors (i.e., cleaning, security, logistic and food service) from manufacturing sector in developed countries like US and Germany; and during this time period there is a significant decline of occupations and value added share in manufacturing sector. Therefore the important contribution of this model is that, it provides a simple mechanism in equation (33) to highlight the additionally independent role of intermediate input outsourcing in structural change.

In addition to structural change implication, the competitive equilibrium result of this model has significant implication on endogenous input-output linkage. This result is summarised in the following proposition

**Proposition 3** (Endogenous Input-Output Linkage and Sectoral Prices): The intermediate input outsourcing share of sector $i$, from sector $j$ is endogenous on all possible sector-to-sector sourcing cost, all possible sectoral specific TFP scale parameters and other parameters. Specifically equation of intermediate input outsourcing share is given by

$$\sigma_{it} = \sum_{j=1}^{n} B_{jit} \quad \text{(34)}$$

$$B_{jit} = \frac{P_{ij}X_{ij}X_{ijt}}{P_{it}Q_{it}} X_{ijt} = \left( \frac{P_{ij}}{P_{it}} \right)^{-\theta} \frac{T_{jt}(P_{jt}\tau_{ij})^{-\zeta}}{T_{jt}(P_{jt}\tau_{ij})^{-\zeta} + T_{it}(\tilde{r}_{it}\tau_{iit})^{-\zeta}} \quad \text{(35)}$$

Meanwhile sectoral gross output price is also endogenous on all possible sector-to-sector sourcing cost, all possible sectoral specific TFP scale parameters and other parameters. Equation of sectoral gross output price and sector-to-sector specific intermediate input price are given by the following equations respectively:

$$P_{it} = \left[ \sum_{j=1}^{n} (P_{ij})^{-\theta} \right]^{-\frac{1}{\theta}} \quad \text{(36)}$$

$$P_{ijt} = \frac{\nu}{\nu - 1} \left[ \Gamma \left( \frac{1 - \nu + \zeta}{\zeta} \right) \right]^{-\frac{1}{1 - \nu}} \left[ T_{jt}(P_{jt}\tau_{ij})^{-\zeta} + T_{it}(\tilde{r}_{it}\tau_{iit})^{-\zeta} \right]^{-\frac{1}{\zeta}} \quad \text{(37)}$$

If there is no sectoral intermediate input linkage between $i$ and $j$, it is a special case of infinitely
large size of sourcing cost $\tau_{ijt}$.

The function form of price and intermediate input linkage intensity are consistent with Eaton and Kortum (2002) and Boehm (2015). Proposition 3 suggests that the intermediate input price $P_{ijt}$ inversely depends on sourcing efficiency$^{18}$. Since efficiency increases with TFP scale; decreases with input cost and sourcing cost, it implies that price decreases with all sectoral TFP; increases with all input cost and sourcing cost. The variation parameter $\zeta$ determines how substitutable of production technology between in-house and outsourcing.

The intermediate input linkage intensity equation is consistent with the trade gravity equation in Eaton and Kortum (2002). Equation (35) suggests that intermediate input linkage depends on intermediate input share and outsourcing share of intermediate input. In particular, intermediate input share adjusts at intensive margin, and outsourcing share adjusts at extensive margin. According to equation (35), higher TFP sector outsources more, which corresponds to the absolute advantage argument in Eaton and Kortum (2002). In addition, this absolute advantage is discounted by input cost and outsourcing cost. Equation (35) also suggests that the variation parameter $\zeta$ measures the sensitivity of intermediate input outsourcing to cost. A lower value of $\zeta$ implies higher outsourcing intensity, which corresponds to the comparative advantage argument in Eaton and Kortum (2002). Furthermore, following proposition 3, the relative sectoral price inversely depends on relative sectoral efficiency$^{19}$. Similar, relative home production share equals relatively weighted average of within sectoral home efficiency to intermediate input efficiency$^{20}$.

4.4 Remark

On top of the theoretical implication, the structural change equation (33) has three implications to empirical study. First, without taking into account the intermediate input outsourcing, price effect suffers endogeneity bias in production based structural change. Equation (35) implies that sectoral intermediate input outsourcing share is endogenous on sectoral price. For the production side based structural change as in equation (33), we cannot unbiasedly identify the price effect without controlling intermediate input supply and demand effect. Because there is obvious endogeneity problem in OLS regression estimation. Every time price varies, we should predict

$^{18}$Here we define the sector pair intermediate input efficiency as $\Phi_{ijt} = T_{jt}(P_{jt} \tau_{ijt})^{-\zeta} + T_{it}(\tilde{r}_{it} \tau_{iit})^{-\zeta}$

$^{19}$If we define $\Phi_{it} = \sum_{j=1}^{n} \Phi_{ijt} \xi_{j}$, then we have $P_{it} = \left(\frac{\Phi_{it}}{\Phi_{jyt}}\right)^{-\frac{1}{\xi}}$.

$^{20}$If we define $\Phi_{ijt} = T_{jt}(P_{jt} \tau_{ijt})^{-\zeta}$; $\Phi_{ijt} = T_{it}(\tilde{r}_{it} \tau_{iit})^{-\zeta}$, then we have $\frac{1-\sigma_{ijt}}{1-\sigma_{jyt}} = \frac{\sum_{k=1}^{n} \left(\frac{\Phi_{ikt}}{\Phi_{jyt}}\right)^{\frac{\sigma_{ikt}}{\sigma_{jyt}} \frac{\Phi_{kjt}}{\Phi_{jyt}}}}{\sum_{k=1}^{n} \left(\frac{\Phi_{ikt}}{\Phi_{jyt}}\right)^{\frac{\sigma_{ikt}}{\sigma_{jyt}} \frac{\Phi_{kjt}}{\Phi_{jyt}}}}$.
there are responses of intermediate input outsourcing decision by individual firms. Then the sectoral intermediate input supply and demand effect activates, which will be mixed with price effect. At the end we cannot estimate unbiasedly how much structural change attributes to price effect. The OLS regression estimation of production based structural change such as in Comin, Lashkari and Mestieri (2015), thereby is actually biased.

Second, according to equation (33) we can use OLS regression to estimate the four individual effect independently. Notice that following proposition 3, sectoral price and outsourcing depend on its own sectoral TFP, distortions and other sectoral TFPs and distortions. On the other hand, they are independent on gross output, final output and intermediate input. Therefore we notice the endogeneity problem of sectoral price and intermediate input outsourcing, but also appreciate the rationality of putting price ratio and outsourcing ratio on the right hand side of a simple OLS regression equation following (33). Because our interested variables are only endogenous on exogenous variables, less likely to be affected by unobservable error terms.

We may further worry about the reverse causality problem. For example, rather than the demand of intermediate input outsourcing cause structural change; structural change may also affect the individual firm level demand decision of intermediate input outsourcing on the other hand. People may worry about that the rise of service sector in many countries is likely to generate the advancement of intermediate input demand from service. We can solve this reverse causality problem and other endogeneity problem by using instrumental variable (IV). Given the exogenous change of distortion over time and the dependence of price and outsourcing share on distortion, we can use distortions as IVs to precisely and independently identify the causal effect of sectoral intermediate input supply and demand effect on structural change.

Third, for the identification of income effect, the income variable is different between consumption side based structural change and production side based structural change. This point is clear to see from equation (32) and equation (33). If we use consumption share as dependent variable, the income is aggregate real consumption; on the other hand if we use value added share as dependent variable, the income is aggregate real gross output. They are not the same. Previous literature of structural change (Comin et al. 2015; Herrendorf et al. 2013b) pays no attention to this difference due to ignorance of input-output linkage.

Notice that now we can briefly prove that the first notion of production function (6) is consistent with the second notion of production function (7). Proposition 1 and proposition 3 hold even without (6). As long as the model setup of the second notion of production holds, it is equivalent to argue that there is a representative producer which minimizes sectoral total cost subject to the sectoral gross output function as in (6). The reason is as following. In the second
notion of production, we define \( \sigma_{it} \equiv \frac{P_{M,Mt}}{P_{at}Q_{it}} \). It implies that the corresponding sectoral value added could be represented by

\[
wL_{it} + rK_{it} = (1 - \sigma_{it})P_{it}Q_{it}
\]

At the firm level equation (11) implies that the optimal allocation ratio of labour to capital would be always the same across all individual in-house production firms over all time period. Then it also implies that this ratio would holds in sectoral level, specifically we have

\[
wL_{it} = \frac{1 - \alpha}{\alpha} rK_{it} = (1 - \sigma_{it})(1 - \alpha)P_{it}Q_{it}
\]

Accordingly (6) is a sufficient production function to generate the equivalence. The first notion of production with equation (6) is not necessary to this model, but just provides a good angle to help understanding.

5. Quantitative Analysis

This section shows the quantitative importance of intermediate input supply and demand effect on structural change. First we run regression of structural change on intermediate input supply and demand effect, price effect and income effect for 35 major economies over 1995-2007, following the structural change equation (33).

Given the OLS estimated coefficients, we compare the four individual effects to see which effect is dominant. This result is presented in section 5.1. Second in order to structurally estimate the four effects, we calibrate the model in section 5.2. We show the identification strategy and parameter values in detail in this section. Third given all the relevant parameters in hand, we simulate the model. We examine the significance and dominance of the four individual effects, based on two counterfactual studies. This result is illustrated in section 5.3.

5.1 Structural change

Motivated by the fact and mechanism, we use sectoral intermediate input supply multiplier to represent the sectoral weight. Following equation (33), the OLS regression equation of structural change is given by

\[
\log \frac{\eta_{ict}}{\eta_{jct}} = \beta \log \frac{1 - \sigma_{ict}}{1 - \sigma_{jct}} + \kappa \log \frac{\mu_{ict}}{\mu_{jct}} + (1 - \rho) \log \frac{P_{ict}}{P_{jct}} + (\xi_i - \xi_j) \log Q_{ct} + f_c + f_t + e_{ct}
\]
Here $f_c$ and $f_t$ are country fixed effect and year fixed effect respectively. The measurement error and other unobservables are contained in error term $\epsilon_{ict}$, conditional on the sector pair $ij$. We choose manufacturing as the benchmark sector. Therefore the structural change that we choose to estimate are relative value added share of other three sectors to manufacturing. The estimate result is presented in table 2.

The first four columns of table 2 report the estimate for all the sampling countries. Column 1 replicates the traditional price effect and income effect on structural change. It suggests significantly positive price effect and negative income effect. The positive price effect is consistent with prediction in Ngai and Pissarides (2007). That is, when sectors are complement ($\rho < 1$), structural change happens from relatively low price sector to relatively high price sector. The price effect is still significant when we add intermediate input demand and supply effect in column 2. But the size is smaller than column 1, which suggests that part of the estimate of price effect in column 1 actually belongs to the intermediate input effect. In column 2, the intermediate input supply and demand effect are significant at 1 percent level. More importantly, their magnitudes are much larger than price effect. It suggests that value added share based structural change has more response from intermediate input supply and demand effect than price effect.

Column 3 and column 4 show as same estimate as in column 2, but with country and year fixed effect. Even with the fixed effects, structural change still responses more to intermediate input supply and demand effect than price effect. For instance, column 4 suggests that given country and year fixed effect, the elasticity of relative value added share to relative intermediate input demand is 0.887; the elasticity of relative value added share to relative intermediate input supply is 0.686; and the elasticity of relative value added share to relative price is 0.503. The first four columns also suggest that income effect is not always significant, particularly not significant when country and year fixed effects are included in column 4.

The larger elasticity of relative value added share to intermediate input supply and demand holds in column 5 to column 7 of table 2. These three columns show another three estimates with country and year fixed effects. The intermediate input demand effect on the right hand side of OLS regression equation contains relative value added which also enters the left hand side of OLS. This may introduce bias. To account for this, we apply the first order Taylor rule approximation: $\log(1 - \sigma_{ict}) \approx -\epsilon_{ict}$. The estimate result is shown in column 5. Not surprisingly, now the estimate of demand elasticity is different, but the supply and price elasticity do not change too much. We further estimate the OLS regression on developed countries and developing countries in column 6 and column 7 respectively. Again we find that the intermediate input
Dependent Variable: \(\log \frac{\eta_j}{\eta_i}\)

<table>
<thead>
<tr>
<th>Coefficient</th>
<th>Column 1</th>
<th>Column 2</th>
<th>Column 3</th>
<th>Column 4</th>
<th>Column 5</th>
<th>Column 6</th>
<th>Column 7</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\beta)</td>
<td>1.486***</td>
<td>0.896***</td>
<td>0.887***</td>
<td>1.737***</td>
<td>0.970***</td>
<td>0.845***</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.065)</td>
<td>(0.045)</td>
<td>(0.042)</td>
<td>(0.083)</td>
<td>(0.053)</td>
<td>(0.056)</td>
<td></td>
</tr>
<tr>
<td>(\kappa)</td>
<td>1.406***</td>
<td>0.803***</td>
<td>0.686***</td>
<td>0.689***</td>
<td>0.646***</td>
<td>0.802***</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.046)</td>
<td>(0.037)</td>
<td>(0.037)</td>
<td>(0.036)</td>
<td>(0.044)</td>
<td>(0.055)</td>
<td></td>
</tr>
<tr>
<td>(1 - \rho)</td>
<td>0.408***</td>
<td>0.272***</td>
<td>0.472***</td>
<td>0.503***</td>
<td>0.478***</td>
<td>0.547***</td>
<td>0.336***</td>
</tr>
<tr>
<td></td>
<td>(0.028)</td>
<td>(0.026)</td>
<td>(0.030)</td>
<td>(0.029)</td>
<td>(0.029)</td>
<td>(0.043)</td>
<td>(0.037)</td>
</tr>
<tr>
<td>(\epsilon_{OG} - \epsilon_{Manu})</td>
<td>-0.024**</td>
<td>0.020**</td>
<td>-0.073***</td>
<td>-0.059</td>
<td>-0.004</td>
<td>0.682***</td>
<td>0.012</td>
</tr>
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<td></td>
<td>(0.011)</td>
<td>(0.008)</td>
<td>(0.028)</td>
<td>(0.052)</td>
<td>(0.051)</td>
<td>(0.070)</td>
<td>(0.068)</td>
</tr>
<tr>
<td>(\epsilon_{MS} - \epsilon_{Manu})</td>
<td>-0.049***</td>
<td>-0.050***</td>
<td>0.190***</td>
<td>-0.029</td>
<td>0.044</td>
<td>0.452***</td>
<td>0.124**</td>
</tr>
<tr>
<td></td>
<td>(0.009)</td>
<td>(0.008)</td>
<td>(0.022)</td>
<td>(0.041)</td>
<td>(0.040)</td>
<td>(0.059)</td>
<td>(0.054)</td>
</tr>
<tr>
<td>(\epsilon_{NMS} - \epsilon_{Manu})</td>
<td>-0.050***</td>
<td>-0.008</td>
<td>0.073**</td>
<td>-0.097*</td>
<td>-0.011</td>
<td>0.022</td>
<td>0.291***</td>
</tr>
<tr>
<td></td>
<td>(0.009)</td>
<td>(0.007)</td>
<td>(0.030)</td>
<td>(0.064)</td>
<td>(0.055)</td>
<td>(0.068)</td>
<td>(0.086)</td>
</tr>
</tbody>
</table>

Table 2: Estimate of structural change

Demand and supply elasticity are always significant and larger than price elasticity. Column 6 and column 7 also suggest that income elasticity are very different between developed countries and developing countries.

To account for the individual effect on structural change, we decompose the structural change into the four effects. Figure 11 presents the decomposition result under the regression estimate in column 4 of table 2. The horizontal axis is year, and the vertical axis is relative value added share of market service to manufacturing. In figure 11, the blue solid line shows the relative value added share in the data. This is time series pattern of average relative value added share of market service to manufacturing for the 35 major economies. From 1995 to 2007, the rising relative value added share implies structural change from manufacturing to market service on average. The red solid line shows the regression predicted relative value added share with country and year fixed effect. Figure 11 suggests that the prediction of OLS regression is very consistent with
Figure 11: Relative value added share of market service to manufacturing

the actual average data. It implies that our structural model do a very good job to match with structural change fact.

Figure 11 suggests that intermediate input supply effect is the dominant effect on structural change. In order to account for each individual effect, we do counterfactual study. First, suppose the relative intermediate input supply multiplier stays at the beginning year, we simulate the relative value added share given equation (33). The green solid line shows the simulated relative value added share without intermediate input supply effect. We continue this simulation exercise by switching off the price effect, income effect and intermediate input demand effect sequentially. Therefore the black line, red dash line and green dash line represent the simulated relative value added share under the three simulation case. According to figure 11, without supply effect, the gap between the simulated data and the benchmark data is largest. Then it is followed by price effect. The income effect and intermediate input demand effect are trivial, compared with supply effect and price effect.

In conclusion, based on the reduced form OLS estimate, the intermediate input supply effect
dominant the structural change. The price effect is always significant, has slightly smaller effect than supply effect. The intermediate input demand elasticity is large and significant, but the demand effect on structural change is trivial. The income effect, however, is not always significant and is trivial.

5.2 Calibration

In this section, we show the calibration of the structural model. First we show the calibration of consumption side parameters. Second we show the calibration of producer side parameters. Last we show the identification strategy of sectoral TFP scale parameter, sourcing cost parameter and the marginal cost parameter.

For the consumption side parameter, we estimate the elasticity of substitution $\varepsilon$, relative income elasticity of demand $\epsilon_i - \epsilon_j$. Following equation (32), we estimate these parameters by OLS regression

$$\log \frac{C_{it}}{C_{jt}} = -\epsilon \log \frac{P_{it}}{P_{jt}} + (\epsilon_i - \epsilon_j) \log C_t + f_c + f_t + \epsilon_{ct}$$

Notice that here we use relative real consumption rather than nominal consumption. Because the relative price also enters to the right hand side of regression equation. The relative time invariant sectoral weight is absorbed by the country fixed effect. All the coefficients are estimated with both country fixed effect and year fixed effect.

For the production side parameters, we first calibrate the variation parameter $\zeta = 2$. This value is much smaller than that of Eaton and Kortum (2002). However, according to Ruhl et al. (2008), the frequent trade data normally implies smaller value of elasticity\(^{21}\). Moreover, the elasticity of substitution between outsourcing and production in-house should be smaller than elasticity of substitution between cross-country final output. While the final outputs are essentially the same no matter which location to produce, the outsourcing intermediate inputs are usually produced by expertised and skillful professionals rather than new professionals in-house. In this sense, the low value of variation parameter is consistent with recent trade literature\(^{22}\). We calibrate $\nu = 2.8$. This parameter does not affect the structural change estimate since the constant mark-up only enters to the constant value for both relative price and intermediate input linkage. The only condition to satisfy following Eaton and Kortum (2002) is that $\zeta > \nu - 1$. In addition, we normalize no sourcing cost within sectoral outsourcing, such that $\tau_{iit} = 1$. Finally we estimate

\(^{21}\)We use annual input-output linkage and price data. This is more frequent data compared with many trade data as suggested by Ruhl et al. (2008).

\(^{22}\)In Eaton, Kortum, Neiman and Romalis (2016), they also calibrate $\zeta = 2$. 
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Benchmark</th>
<th>Counterfactual 1</th>
<th>Counterfactual 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \zeta )</td>
<td>2</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>( \nu )</td>
<td>2.8</td>
<td>2.8</td>
<td>2.8</td>
</tr>
<tr>
<td>( \theta )</td>
<td>1.652</td>
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<td>1.652</td>
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<tr>
<td>( \varepsilon )</td>
<td>0.344</td>
<td>0.344</td>
<td>0.344</td>
</tr>
<tr>
<td>( \beta )</td>
<td>0.887</td>
<td>0.971</td>
<td>0.975</td>
</tr>
<tr>
<td>( \kappa )</td>
<td>0.686</td>
<td>0.889</td>
<td>1.186</td>
</tr>
<tr>
<td>( \rho )</td>
<td>0.497</td>
<td>0.687</td>
<td>0.528</td>
</tr>
<tr>
<td>( \epsilon_{OG} - \epsilon_{Manu} )</td>
<td>0.004</td>
<td>0.004</td>
<td>0.004</td>
</tr>
<tr>
<td>( \epsilon_{MS} - \epsilon_{Manu} )</td>
<td>0.004</td>
<td>0.004</td>
<td>0.004</td>
</tr>
<tr>
<td>( \epsilon_{NMS} - \epsilon_{Manu} )</td>
<td>-0.316</td>
<td>-0.316</td>
<td>-0.316</td>
</tr>
<tr>
<td>( \xi_{OG} - \xi_{Manu} )</td>
<td>-0.059</td>
<td>0.046</td>
<td>-0.008</td>
</tr>
<tr>
<td>( \xi_{MS} - \xi_{Manu} )</td>
<td>-0.029</td>
<td>0.031</td>
<td>-0.007</td>
</tr>
<tr>
<td>( \xi_{NMS} - \xi_{Manu} )</td>
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<td>0.002</td>
<td>-0.093</td>
</tr>
<tr>
<td>( \tau_{ijt} )</td>
<td>( \hat{\tau}_{ijt} )</td>
<td>( \hat{\tau}_{ijt,1995} )</td>
<td>( \hat{\tau}_{ijt} )</td>
</tr>
<tr>
<td>( T_{it} )</td>
<td>( \hat{T}_{it} )</td>
<td>( \hat{T}_{it} )</td>
<td>( \hat{T}_{i,1995} )</td>
</tr>
</tbody>
</table>

Table 3: Parameter calibration result

the value of \( \theta \) from the relative within sectoral outsourcing share equation

\[
\frac{B_{it}}{B_{jlt}} = \left( \frac{A_{it}^{V,A}}{A_{jt}^{V,A}} \right)^{\theta} \left( \frac{P_{it}}{P_{jt}} \right)^{\theta-\zeta} \left[ \frac{P_{jt}^{-\zeta} + \tilde{r}_{jt}^{-\zeta}}{P_{it}^{-\zeta} + \tilde{r}_{it}^{-\zeta}} \right]^{1-\theta-\zeta}
\]

Here \( A_{it}^{V,A} \) is sectoral value added based TFP, which is estimated following Hu (2017). In addition, the implicit assumption in the relative within sectoral outsourcing share equation is that, \( A_{it}^{V,A} \) is the mean value of Frechet distribution \( F_{it}(a) \)\(^{23}\).

For the identification of outsourcing cost \( \tau_{ijt} \), sectoral TFP parameter \( T_{it} \) and marginal cost parameter \( \tilde{r}_{it} \), we use the non-linear fixed effect method following proposition 3. Specifically for every country at every year, we have 20 values to estimate\(^{24}\). Given the data \( B_{jlt} \) and \( P_{it} \), the 20

\(^{23}\)If this assumption holds, we have \( T_{it} = \left( \frac{A_{it}^{V,A}}{1+(1-1/\zeta)} \right)^{\zeta} \).

\(^{24}\)Since we have 4 sectors, we have 4 parameters to estimate for \( T_{it} \) and \( \tilde{r}_{it} \) respectively. In addition, given the normalization of \( \tau_{it} = 1 \), now we have 12 parameters to estimate for \( \tau_{ijt} \).
values are exactly identified. The 20 values are identified by the following recursive fixed effect procedure.

1. Given \( \tilde{\tau}_{it} \), \( T_{it} \) are estimated from \( B_{it} \) equation (35) and (37).
2. Given \( \tilde{r}_{it} \) and \( T_{it} \), \( \tau_{ijt} \) are estimated from \( B_{jit} \) equation (35) and (37).
3. Given \( T_{it} \) and \( \tau_{ijt} \), \( \tilde{r}_{it} \) are estimated from \( P_{it} \) equation (36) and (37).
4. Repeat the first three steps until \( T_{it} \) converge.

This method is extended to every country at every year. The calibration result of benchmark case is presented in column 2 of table 3.

5.3 Counterfactual Study

We simulate two counterfactual cases in this section. In the first counterfactual study, we assume that the outsourcing cost \( \tau_{ijt} \) stays at the first year level for all sectors and for all countries. In the second counterfactual study, we assume that the sectoral TFP scale parameter \( T_{it} \) stays at the
first year level for all sectors and for all countries. In both cases, we use the structural model to simulate the value added share, price, intermediate input linkage intensity. Given the simulated data, we use OLS regression to estimate the four individual effects in the same way as in section 5.1. Then given the new estimated elasticities, we decompose the four individual effects and compare them as in section 5.1. The objective of these two counterfactual studies is that we want to see whether the decomposition result in section 5.1 is still valid given our structural model based simulation.

The parameter calibration result under the two counterfactual cases are presented in column 3 and column 4 of table 3. In column 3, we assume $\tau_{ijt} = \tilde{\tau}_{ij,1995}$, and assume other parameters do not change except for structural change parameters. Then all the individual structural change parameters are re-estimated given model simulated data. The column 3 suggests that the structural change parameters do not change too much. The elasticity of supply, demand and price are slightly larger than the benchmark calibration result; and the relative income elasticities are still trivial. In column 4, we assume $T_{it} = \tilde{T}_{i,1995}$ and the rest of study just follows the counterfactual
case 1. We find similar result as in counterfactual case 1.

Following the counterfactual estimate, we decompose the four individual effects in figure 12 under counterfactual case 1; and in figure 13 under counterfactual case 2. Both figures suggest that the intermediate input supply effect is the dominant effect on structural change. While price effect is significantly large, it is much smaller than intermediate input supply effect. According to the result, intermediate input supply effect is more than twice as large as the price effect. In addition, the intermediate input demand effect and income effect are trivial, compared with the two aforementioned effects. This result is consistent with the decomposition result in section 5.1.

6. Conclusion

This research highlights the importance of input-output linkage in affecting structural change. If we ignore the input-output linkage, structural change from manufacturing to service are understood mainly due to two factors: relative low price of manufacturing to service; relative low income elasticity of manufacturing to service. Given input-output linkage, we identify another mechanism. Structural change happens from relative more intermediate input demandable sector to more intermediate input suppliable sector. This mechanism suggests that since manufacturing sector start to outsources more and more intermediate input to service sector, labour and capital shift out from manufacturing to service. Manufacturing becomes to relatively intermediate input demandable sector; service becomes to relatively intermediate input suppliable sector. Therefore, the most important contribution of this research is that we find the role of input-output linkage in structural change.

A natural question follows the mechanism of intermediate input outsourcing is that how important is this mechanism? To answer this positive question well, we build up a general equilibrium model. We model the inter-sectoral trade of intermediate input in a similar way to the international trade of final output following the Ricardian trade literature. We allow nonhomothetic CES preference in sectoral consumption. We calibrate the model and then simulate the model. The quantitative analysis suggests that intermediate input supply effect dominant the structural change. The price effect is significant, but smaller than intermediate input supply effect. Income effect and intermediate input demand effect are generally trivial.

This research has significant implication in the role of input-output linkage in development study. We all know that countries are connected by international trade, and international trade has profound implication on income difference, productivity difference, capital mobility and trade balance and so on. Similarly, within the country, sectors and firms are not independent to
each other. They are connected by input-output linkage or network. There are many good reasons to expect the linkage or network effect on development, for instance, on structural change in this research. More importantly, this domestic linkage effect may be much more important than we expect. Three implications for the future study are worth to notice. First, input-output linkage is likely to have effect on other development studies and broadly have effect on macroeconomic studies. Second, at firm level this is input-output network. Given the importance role of heterogeneous firm in new classical macroeconomic model, input-output network probably is another important element. Third, domestic input-output linkage is extended to be global offshoring and international outsourcing. Given both importance of domestic input-output linkage and global offshoring in literature, the relationship between them are actually less known.

A. Appendix: Proofs of the Propositions

Proof of Proposition 1. Equilibrium Solution of Structure Terms

We can start with consumer’s problem. The two constraint equations (3) and (4) are transformed to become

$$\sum_{i=1}^{n} \Omega_{it}^{1/\varepsilon} \left( \frac{C_{it}}{C_{t}^{\varepsilon-1}} \right)^{\varepsilon-1} = 1 \quad (A.1)$$

$$\sum_{i=1}^{n} P_{it} C_{it} = E_t; \text{ where } E_t = w_t L_t + (1 + r_t) A_t - A_{t+1} \quad (A.2)$$

The Lagrangian equation is given by

$$\mathcal{L} = C_t + \rho_t \left[ 1 - \sum_{i=1}^{n} \Omega_{it}^{1/\varepsilon} \left( \frac{C_{it}}{C_{t}^{\varepsilon-1}} \right)^{\varepsilon-1} \right] + \omega_t \left( E_t - \sum_{i=1}^{n} P_{it} C_{it} \right)$$

The corresponding first order condition is given by

$$P_{it} C_{it} = \frac{\rho_t}{\omega_t} \left( 1 - \varepsilon \right) \Omega_{it}^{1/\varepsilon} \left( \frac{C_{it}}{C_{t}^{\varepsilon-1}} \right)^{\varepsilon-1} \quad (A.3)$$

Given equation (A.3), we have the following equation:

$$\sum_{i=1}^{n} P_{it} C_{it} = \frac{\rho_t}{\omega_t} \left( 1 - \varepsilon \right) \sum_{i=1}^{n} \Omega_{it}^{1/\varepsilon} \left( \frac{C_{it}}{C_{t}^{\varepsilon-1}} \right)^{\varepsilon-1} = \frac{\rho_t}{\omega_t} \left( 1 - \varepsilon \right)$$

The second equality holds by applying the fact from equation (A.1). Define $P_{t} C_{t} = \sum_{i=1}^{n} P_{it} C_{it}$,
then we have

\[ P_t C_t = \frac{\rho_t}{\omega_t} \frac{1 - \varepsilon}{\varepsilon} \]  

(A.4)

Substitute equation (A.4) to equation (A.3), we have

\[ \frac{P_t C_{it}}{P_t C_t} = \Omega_t^\frac{1}{1-\varepsilon} \left( \frac{C_{it}}{C_{i,t}^{1-\varepsilon}} \right)^{\frac{\varepsilon-1}{\varepsilon}} \]  

The therefore the optimal intratemporal sectoral consumption allocation \( C_{it} \) is solved as in equation (28)

Based on equation (28), we can derive the consumption share as in equation (27). Then given equation (27), we have the following:

\[ \sum_{i=1}^{n} \frac{P_t C_{it}}{P_t C_t} = \sum_{i=1}^{n} \Omega_i \left( \frac{P_t}{P_t} \right)^{1-\varepsilon} C_{i,t}^{\varepsilon-1} = 1 \]

Accordingly the aggregate price index is derived as in equation (30).

Now we turn to the production's problem at sectoral level. The derivation of sectoral gross output equation (29) and the corresponding aggregate price index equation (31) follows exactly the same way as solving the optimal sectoral consumption and aggregate price above. Now we could think of a social planner which maximizes intratemporal aggregate gross output at every time period \( t \).

We denote the sector-to-sector ratio of intermediate input outsourcing to gross output as matrix \( B \), such that \( B_{jit} \equiv \frac{P_{X_j X_i}}{P_{it} Q_{it}} \). Therefore the budget constraint equation (16) becomes to

\[ P_{jt} C_{jt} + \sum_{i=1}^{n} B_{jit} P_{it} Q_{it} = P_{jt} Q_{jt} \]  

(A.5)

Divide both sides of the equation (A.5) by nominal GDP or aggregate value added \( P_{Y_t} Y_t \), the budget constraint could be transformed by the following equation.

\[ \gamma_{jt} \equiv \frac{P_{jt} Q_{jt}}{P_{Y_t} Y_t} = \frac{P_{jt} C_{jt}}{P_{Y_t} Y_t} + \sum_{i=1}^{n} \frac{B_{jit} P_{it} Q_{it}}{P_{Y_t} Y_t} \]

Following the equation above, we can further derive budget constraint as

\[ \gamma_t = \lambda_t + B_t \gamma_t \]  

(A.6)

Then we can easily solve equation (A.6), and get the vector result of Domar weight as in equation
Given the solution of Domar weight, the value added share is simply solved by following the fact that \( \eta_{it} = (1 - \sigma_{it}) \gamma_{it} \), where \( \sigma_{it} \equiv \sum_{j=1}^{n} \frac{P_{ijt} X_{ijt}}{P_{it} Q_{it}} \).

Assume there is a representative producer which maximizes the sectoral gross profit by optimally allocate sectoral labour and capital. Given the Cobb-Douglas assumption of sectoral gross output equation (6), the first order condition of capital and labour are given by

\[
(1 - \sigma_{it}) \alpha \frac{P_{it} Q_{it}}{K_{it}} = r \quad (A.7)
\]

\[
(1 - \sigma_{it})(1 - \alpha) \frac{P_{it} Q_{it}}{L_{it}} = w \quad (A.8)
\]

Equation (A.7) and (A.8), together with the fact that \( \sum_{i=1}^{n} (1 - \sigma_{it}) \gamma_{it} = 1 \) implies that

\[
K_{it} = (1 - \sigma_{it}) \gamma_{it} K_t \quad (A.9)
\]

\[
L_{it} = (1 - \sigma_{it}) \gamma_{it} L_t \quad (A.10)
\]

Therefore the employment share is solved as in equation (25). Notice that the solution of employment share does not depend on the production function (6). It could be solved by simply follow the second notion of production. For more details please see the comment following proposition 3.

QED.

**Proof of Proposition 3. Endogenous Intermediate Input Share and Sectoral Prices**

The probability of in-house price larger and equal to a constant value \( p \) is given by

\[
\Pr(P_{ijt}^H(\omega) \geq p) = \Pr\left(\frac{\nu}{\nu - 1} \frac{\bar{r}}{a_{ijt}^H(\omega)} \geq p\right)
= \Pr\left(\frac{a_{ijt}^H(\omega)}{\nu - \frac{\bar{r}}{p}} \leq \frac{\nu}{\nu - 1} \right)
\]

The above inequality together with equation (13) implies that the CDF of in-house price is derived by

\[
G_{ijt}(p) \equiv \Pr(P_{ijt}^H(\omega) \leq p) = 1 - \exp\left[-A_{it} \left(\frac{\nu}{\nu - 1} \frac{\bar{r}}{p}\right)^{-\zeta}\right] \quad (A.11)
\]

Similar the CDF of outsourcing price from sector \( j \) at time period \( t \) is derived as

\[
G_{ijt}(p) \equiv \Pr(P_{ijt}^X(\omega) \leq p) = 1 - \exp\left[-A_{jt} \left(\frac{\nu}{\nu - 1} \frac{\tau_{ijt} P_{jt}}{p}\right)^{-\zeta}\right] \quad (A.12)
\]
According to equation (15), the probability of intermediate input variety price which is not larger than $p$ the producer of sector $i$ actually pay is given by

$$
\Pr(P^*_{ijt}(\omega) \leq p) = 1 - \Pr(P^*_{ijt}(\omega) \geq p) = 1 - \Pr(P^H_{ijt}(\omega) \geq p, P^X_{ijt}(\omega) \geq p) = 1 - (1 - \Pr(P^H_{ijt}(\omega) \leq p))(1 - \Pr(P^X_{ijt}(\omega) \leq p))
$$

Substitute equation (A.11) and equation (A.12) to the above inequality, we can derive the CDF of intermediate input variety price producer of sector $i$ actually pay, which is given by

$$
G_{ijt}(p) \equiv \Pr(P^*_{ijt}(\omega) \leq p) = 1 - \exp(-\Phi_{ijt}p^\zeta)
$$

Here $\Phi_{ijt} = \left(\frac{\nu}{\nu-1}\right)^{-\zeta}(A_{it}\tilde{r}^{-\zeta} + A_{jt}(\tau_{ijt} P_{jt})^{-\zeta})$.

Let $\pi_{ijt}(\omega)$ be the unconditional probability of variety $\omega$ which is sourced from sector $j$ to sector $i$. This probability is solved as following:

$$
\pi_{ijt}(\omega) = \Pr(P^X_{ijt}(\omega) \leq P^H_{ijt}(\omega)) = \int_0^\infty \exp\left[-A_{it}\left(\frac{\tilde{r}}{p}\right)^{-\zeta}\left(\frac{\nu}{\nu-1}\right)^{-\zeta}\right] dG_{ijt}(p)
$$

$$
= \int_0^\infty \left(\frac{\nu}{\nu-1}\right)^{-\zeta} A_{jt}(\tau_{ijt} P_{jt})^{-\zeta} \frac{1}{\Phi_{ijt}} \int_0^\infty \tilde{r}^{-\zeta-1} \phi_{ijt}(\tilde{r}) \exp(-\Phi_{ijt}p^\zeta) dp
$$

$$
= \frac{A_{jt}(\tau_{ijt} P_{jt})^{-\zeta}}{A_{it}\tilde{r}^{-\zeta} + A_{jt}(\tau_{ijt} P_{jt})^{-\zeta}}
$$

(A.14)

This is the probability that sector $i$ outsources from sector $j$ with any randomly chosen variety $\omega$. This is also the fraction of $\omega \in [0, 1]$ which sector $i$ outsources from sector $j$ rather than produce in house. Moreover equation (A.13) also implies that the probability does not depend on intermediate input variety, that is, equation (A.13) suggests that

$$
\pi_{ijt}(\omega) = \pi_{ijt}
$$

The conditional CDF of price that producer of sector $i$ outsources from producer of sector $j$
is given by

\[
G_{ijt}(p) = \Pr(P_{ijt}(\omega) \leq p | P_{ijt}(\omega) \leq P_{ijt}^H(\omega))
\]

\[
= \frac{1}{\pi_{ijt}} \int_0^p \Pr(z \leq P_{ijt}^H(\omega)) dG_{ijt}(z)
\]

\[
= \frac{1}{\pi_{ijt}} \int_0^p \exp \left[ -A_{it} \left( \frac{\bar{r}}{z} \right)^{-\zeta} \left( \frac{\nu}{\nu - 1} \right)^{-\zeta} \right] dG_{ijt}(z)
\]

\[
= \frac{1}{\pi_{ijt}} \int_0^p \left( \frac{\nu}{\nu - 1} \right)^{-\zeta} A_{it}(\tau_{ijt}P_{ijt})^{-\zeta} \zeta z^{\zeta-1} \exp(-\Phi_{ijt} z^\zeta) dz
\]

\[
= \left( \frac{\nu}{\nu - 1} \right)^{-\zeta} A_{it}(\tau_{ijt}P_{ijt})^{-\zeta} \frac{1}{\Phi_{ijt}} \int_0^p \zeta z^{\zeta-1} \Phi_{ijt} \exp(-\Phi_{ijt} z^\zeta) dz
\]

\[
= \int_0^p \zeta z^{\zeta-1} \Phi_{ijt} \exp(-\Phi_{ijt} z^\zeta) dz
\]

\[
= \Pr(P_{ijt}^*(\omega) \leq p)
\]

\[
= G_{ijt}(p)
\]  

Therefore the distribution of intermediate input outsourcing price of sector i from sector j is exactly the same as the general price distribution in sector i.

Assume the representative producer minimizes the total intermediate input cost at every sector pair ij, subject to the intermediate input pair equation (8). Given the aggregate intermediate input of sector i from sector j is a CES sum of individual intermediate input variety as in equation (8), the equilibrium quantity and aggregate price of individual intermediate input variety are derived by the following two equations.

\[
X_{ijt}(\omega) = \left( \frac{P_{ijt}(\omega)}{P_{ijt}} \right)^{-\nu} X_{ijt}
\]  

\[
P_{ijt} = \left( \int_0^1 P_{ijt}(\omega)^{1-\nu} d\omega \right)^{-\frac{1}{1-\nu}}
\]

Based on equation through (A.15) to (A.17), the total spending of outsourcing of sector i from
sector \( j \) at time period \( t \) is given by

\[
P_{ijt}X_{ijt} = \int_0^1 P_{ijt}(\omega)X_{ijt}(\omega)\,d\omega
\]

\[
= \int_{\{P_{ijt}(\omega) = P_{ijt}^*(\omega)\}} P_{ijt}^*(\omega)X_{ijt}(\omega)\,d\omega
\]

\[
= P_{ijt}^{-\nu}P_{ijt}X_{ijt}\int_{\{P_{ijt}(\omega) = P_{ijt}^*(\omega)\}} P_{ijt}^*(1-\nu)(\omega)\,d\omega
\]

\[
= P_{ijt}^{-\nu}P_{ijt}X_{ijt}\left[P_{ijt}^*(1-\nu)(\omega)\Pr(\omega \in \{P_{ijt}(\omega) = P_{ijt}^*(\omega)\})\right]
\]

\[
= P_{ijt}^{-\nu}P_{ijt}X_{ijt}\pi_{ijt}
\]

\[
= P_{ijt}X_{ijt}\pi_{ijt}
\] (A.18)

Equation (A.18) implies that the outsourcing share of intermediate input of sector \( i \) from sector \( j \) equals the unconditional probability of outsourcing variety from sector \( j \) by sector \( i \). That is,

\[
\frac{P_{ijt}X_{ijt}}{P_{ijt}X_{ijt}} = \pi_{ijt} = \frac{A_{ji}(\tau_{ijt}P_{jt})^{-\zeta}}{A_{it}^{-\zeta} + A_{ji}(\tau_{ijt}P_{jt})^{-\zeta}}
\] (A.19)

Assume the representative producer of every sector also minimizes the total matrix cost or total intermediate input cost from all other sectors, subject to the sectoral gross output function (7). Given the CES sum of sectoral gross output, the equilibrium quantity of intermediate input is given by

\[
X_{ijt} = \left(\frac{P_{ijt}}{P_{it}}\right)^{-(1+\theta)}Q_{it}
\] (A.20)

The corresponding equilibrium price of intermediate input is given by equation (??). Equation (A.21) implies that the equilibrium share of intermediate input of sectoral pair \( ij \) to sector \( i \) is given by

\[
\frac{P_{ijt}X_{ijt}}{P_{it}Q_{it}} = \left(\frac{P_{ijt}}{P_{it}}\right)^{-\theta}
\] (A.21)

Therefore equation (A.19) and equation (A.21) together imply that equation (35) holds in equilibrium. Then given equation (35) and the definition (24), the equation (??) holds in equilibrium as well.

We can derive the intermediate input price \( P_{ijt} \) by following equation (A.17) and the CDF
formula of $P_{ijt}$ from equation (A.13). Specifically we have the following solution:

$$P_{ijt}^{1-\nu} = \int_0^1 P_{ijt}(\omega)^{1-\nu} d\omega = \int_0^\infty p^{1-\nu} dG_{ijt}(p) = \int_0^\infty p^{1-\nu} \Phi_{ijt} p\zeta^{-1} \exp(-\Phi_{ijt} p\zeta) dp$$

Denote $z = \Phi_{ijt} p\zeta$, then we have $dz = \Phi_{ijt} \zeta p^{-1} dp$. Use this definition, we can further solve $P_{ijt}$ by following the above equation.

$$P_{ijt}^{1-\nu} = \int_0^\infty p^{1-\nu} \exp(-\Phi_{ijt} p\zeta) dz = \int_0^\infty \left( \frac{z}{\Phi_{ijt}} \right)^{1-\nu} \exp(-z) dz = \Phi_{ijt}^{\frac{1-\nu}{\zeta}} \int_0^\infty z^{1-\nu+1} \exp(-z) dz = \Phi_{ijt}^{\frac{1-\nu}{\zeta}} \Gamma \left( \frac{1-\nu}{\zeta} + 1 \right)$$  \hspace{1cm} (A.22)

Equation (A.22) with definition of $\Phi_{ijt}$ from equation (A.13) implies that equation (??) holds in equilibrium.

QED.

**Proof of Proposition 2. Structural Transformation**

Equation (32) follows the first difference of logarithm transformation of equation (28). Make a transformation of equation (28) and (29), we have the following two equations:

$$\Delta \log \frac{C_{it}}{C_{jt}} = \Delta \log \frac{\Omega_i}{\Omega_j} - \epsilon \Delta \log \frac{P_{it}}{P_{jt}} + \Delta \log \frac{C_{it}^{\epsilon_i}}{C_{jt}^{\epsilon_j}}$$  \hspace{1cm} (A.23)

$$\Delta \log \frac{Q_{it}}{Q_{jt}} = \Delta \log \frac{\Omega_i}{\Omega_j} - \epsilon \Delta \log \frac{P_{it}}{P_{jt}} + \Delta \log \frac{Q_{it}^{\epsilon_i}}{Q_{jt}^{\epsilon_j}}$$  \hspace{1cm} (A.24)

Based on equation (A.23) and equation (A.24), we have the following equation:

$$\Delta \log \frac{Q_{it}}{Q_{jt}} - \Delta \log \frac{C_{it}}{C_{jt}} = (\epsilon_i - \epsilon_j) \Delta \log Q_t - (\epsilon_i - \epsilon_j) \Delta \log C_t$$  \hspace{1cm} (A.25)

Equation (A.25) implies that

$$\Delta \log \frac{P_{it}Q_{it}}{P_{jt}Q_{jt}} - \Delta \log \frac{P_{it}C_{it}}{P_{jt}C_{jt}} = (\epsilon_i - \epsilon_j) \Delta \log Q_t - (\epsilon_i - \epsilon_j) \Delta \log C_t$$
We can then further transform the above equation into

\[ \Delta \log \frac{\gamma_{it}}{\gamma_{jt}} - \Delta \log \frac{\lambda_{it}}{\lambda_{jt}} = (\epsilon_i - \epsilon_j) \Delta \log Q_t - (\epsilon_i - \epsilon_j) \Delta \log C_t \]

Equation (25) implies that we can transform the first difference of employment share and value added share in logarithm as following

\[ \Delta \log \frac{l_{it}}{l_{jt}} = \Delta \log \frac{\eta_{it}}{\eta_{jt}} = \Delta \log \frac{1 - \sigma_{it}}{1 - \sigma_{jt}} + \Delta \log \frac{\gamma_{it}}{\gamma_{jt}} \]

\[ = \Delta \log \frac{1 - \sigma_{it}}{1 - \sigma_{jt}} + \Delta \log \frac{\lambda_{it}}{\lambda_{jt}} + (\epsilon_i - \epsilon_j) \Delta \log Q_t - (\epsilon_i - \epsilon_j) \Delta \log C_t \]

\[ = \Delta \log \frac{1 - \sigma_{it}}{1 - \sigma_{jt}} + (1 - \epsilon) \Delta \log \frac{P_{it}}{P_{jt}} + (\epsilon_i - \epsilon_j) \Delta \log Q_t \]

Therefore we can arrive at equation (33). QED.

**Proof of Proposition ??. Identification of Sectoral Intermediate Input Outsourcing Distortion**

Define \( \kappa_{ijt} = \sigma_{it} \left( \frac{P_{jt}}{P_{Mit}} \right)^{-\theta} \). Equation (35) is rewritten as

\[ B_{ijt} = \frac{\kappa_{ijt} A_{jt} (P_{jt} \tau_{ijt})^{-\zeta}}{A_{jt} (P_{jt} \tau_{ijt})^{-\zeta} + A_{it} \tilde{r}^{-\zeta}} \]

We can further transform this equation to become

\[ \frac{1}{B_{ijt}} = \frac{1}{\kappa_{ijt}} + \frac{1}{\kappa_{ijt} A_{jt}} \left( \frac{\tilde{r}}{P_{jt} \tau_{ijt}} \right)^{-\zeta} \]

Assume there is no within sectoral distortion of intermediate input outsourcing, the above equation implies that the following two accounting equations hold:

\[ \frac{1}{B_{ijt}} - \frac{1}{\kappa_{ijt}} = \frac{1}{\kappa_{ijt} A_{jt}} \left( \frac{\tilde{r}}{P_{jt} \tau_{ijt}} \right)^{-\zeta} \quad \text{(A.26)} \]

\[ \frac{1}{B_{jjt}} - \frac{1}{\kappa_{jjt}} = \frac{1}{\kappa_{jjt} A_{jt}} \left( \frac{\tilde{r}}{P_{jt}} \right)^{-\zeta} \quad \text{(A.27)} \]

We can solve \( \tau_{ijt} \) by following equation (A.26) and equation (A.27). Then substitute the definition of \( \kappa_{ijt} \), the equation of \( \tau_{ijt} \) is solved as in equation (??). QED.
References


