

Closing the Eyes on a Gloomy Future: Psychological Causes and Economic Consequences

Rachid Laajaj*

December 7, 2011

“My third maxim was to endeavor always to conquer myself rather than fortune, and change my desires rather than the order of the world”

Rene Descartes, Discours de la Méthode (1637), III, p. 595 596, Translated by John Veitch 1901

Abstract

This paper analyzes how vulnerability reduces the time horizon of the poor facing gloomy economic prospects. Research in psychology has shown how individuals modify their preferences in order to reduce their cognitive dissonance, which is the uncomfortable tension felt when simultaneously holding conflicting thoughts. It occurs among the poor when simultaneously caring about their future welfare while having gloomy economic prospects. Hence closing their eyes on the future can reduce their psychological distress at the cost of worsening their future economic wellbeing. This paper offers a new theoretical approach that decomposes time discounting and analyzes the endogenous determination of the individual's time horizon. The model predicts that the time horizon of a poor individual is increasing in initial wealth, resulting in a behavioral poverty. The prediction is tested using data from a randomized experiment in Mozambique, which confirms that the beneficiaries of an agro-input subsidy and a matched savings intervention increased their planning horizon as a result of their improved economic prospects.

JEL Codes: D91, O12, Q12

*University of Wisconsin Madison, Paris School of Economics. I am grateful to Michael Carter, Dean Yang, Laura Schechter, Karen Macours, Bradford Barham, Travis Lybbert, Dylan Fitz, Elizabeth Tremmel and seminar participants at Paris School of Economics and UC Davis for helpful discussions and suggestions. I particularly thank Aniceto Matias and Ines Vilela for an outstanding supervision of data collection and data entry. I am grateful to IFDC Mozambique for hosting this project. This research was made possible, in part, through support provided by the U.S. Agency for International Development through the BASIS Assets and Market Access Collaborative Research Support Program.

1 Endogenous time discounting, cognitive dissonance and anticipatory utility

Since Rae (1834:109) stated that what he called the “effective desire of capital accumulation” is the psychological factor that most affects the growth of countries, numerous studies have confirmed the positive correlation between patience (or time discounting) and income. Subsequently, the pioneer work of Strotz (1955) and Pollak (1968) led to a burgeoning literature on inconsistent time preference and its consequence on long term well-being (e.g. Laibson 1997, O’Donoghue and Rabin and Banerjee and Mullainathan 2010), which inspired interventions such as commitment savings (Ashraf et al. 2005, Brune et al. 2011) qualified by Loewenstein et al. (2008) as “light paternalism”. Yet without a deeper understanding of the causes of the reluctance or difficulties for the poor to save, as observed by Duflo and Banerjee (2007), interventions may address the symptom rather than the source of the issue, which limits their chances of generating a life change in the economic path of the beneficiaries. Becker and Mulligan (1997) are among the rare publications investigating the possibility of the endogenous determination of the time preference, with a model where the individual can invest in a “forward looking capital” in order to build patience, they find that the discount factor is increasing in the initial wealth of the individual.

Another body of the literature looks at aspirations and fatalism (Macours and Vakis 2009, Bernard et al. 2011, Ray 2006, Appadurai 2004). Macours and Vakis (2009) find that the exogenous participation of community leaders in an asset transfers program increased the recipients’ aspirations as well as their investment in human and physical capital. In an experiment in Nicaragua, According to a beneficiary’s own words, “Before the program, I just thought about working in order to eat from day to day. Now I think about working in order to move forward through my business”. Bernard et al. (2011) find evidence of fatalistic beliefs among rural households, meaning that they believe that they have little control over their future. In both cases, aspirations are highly interrelated with the time horizon of the individual and the authors find that aspirations are endogenously determined and matter for long term economic outcomes.

This paper investigates theoretically and empirically the endogenous determination of an individual’s time horizon. The time horizon is defined as the extent to which the individual identifies with future selves at any given point in time. The individual will feel some utility from anticipation and plan his expenditures only for periods that are within his time horizon. I also present a version of the model where the time horizon fades away progressively. In this case, the horizon function indicates the extent to which an individual relates to his future self, which is a decreasing function of the time span between the present and a given future period. The horizon function closely relates to the recent psychology literature, which indicates decision making involving future selves is similar to the one involving other individuals (Jamison and Wegener, 2010), and that the *psychological remoteness* determines the level of altruism (Trout, 2007). As the time span increases between now and a perceived future, the *psychological remoteness* toward this future self increases which reduces the altruism toward this future self decreases, affecting both the planning of consumption and the utility from anticipation. The utility from anticipation worsens the condition of the poor, who faces a dilemma caused by his gloomy prospects and his desire to have a better future. The dilemma causes a cognitive dissonance, which can be lessened by a reduction of the time horizon, which in turn makes the poor more vulnerable in the long turn, generating a behavioral poverty trap.

The prediction of the model are tested over 1,546 rural households in the Manica province of Mozambique. I find that randomly selected recipients of either an agro-input subsidy or a matched savings intervention, reacted to the intervention by increasing their planning horizon by two to three months (an increase of 30 to 50% of the initial time horizon). As predicted, I also find that this effect is economically and statistically more significant for the small

farmers than for the large ones. The results highlight that besides its direct economic impact, the interventions also had a substantial behavioral impact. I argue for a more careful look at the behavioral impact of interventions, which can have a multiplier effect and a potentially high influence on long term poverty.

1.1 Cognitive dissonance and endogenous time discounting

Since Festinger (1957), cognitive dissonance has been one of the most influential theories in social psychology. It was introduced into economics by Hirschman (1965) and was first modeled by Akerlof and Dickens (1982). Cognitive dissonance is the feeling of uncomfortable tension which comes from holding conflicting thoughts in the mind at the same time. As a consequence, the individual mind alters his beliefs or preferences in a way that reduces this dissonance. Bramel (1962), relates cognitive dissonance with ego-defensive processes, showing that in some situations, an individual can attribute undesirable characteristics to other people in order to preserve his self-esteem. In a well known illustration of this mechanism, two years later Glass (1964) found that students who were asked to give electric shocks to victims tend to attribute undesirable characteristics to the victims in order to preserve their self-esteem.

Leibow (1967)'s classic ethnographic book is revisited by Montgomery (1994) to analyze the cognitive dissonance of the street-corner man. The contradiction between the social values that dictate that a man must financially support his family and inability of the street-corner man to do so, generates a cognitive dissonance, that causes major distress. In Montgomery's model of endogenous altruism, a man receives a positive utility from supporting the members of his family, but a negative utility from the cognitive dissonance, both proportional to his attachment to his family. Hence, through an unconscious process, the street-corner man detaches himself from his family to limit this cognitive dissonance.

Reproducing the reasoning of Montgomery (1994) in an intertemporal perspective, the poor face three conflicting thoughts:

- 1- I am a good and successful person
- 2- I want to reach decent living conditions in the future¹
- 3- I am unable to ensure myself (and my family) a decent future

The three thoughts are conflicting because the inability to reach one's objective is a failure, which questions the first thought. Hence the ego-defensive process of the individual needs to alter either the second or the third statement. A first set of less conflicting thoughts consists in replacing the third statement by:

- 3- I will work hard to ensure myself (and my family) a decent future

However, when the prospective are very gloomy and the third statement is not achievable or comes at a dissuasive utility cost, then an alternative scenario is to replace the second statement by:

- 2- I live my life day after day without worrying about the future

Hence the poor can either face anxiety and low self-esteem, or close their eyes on the future, which harms their long term economic development. The quotes from Narayan and Ebrary (2000) in the introduction seems to indicate

¹Notice that the individual's perception of a decent living condition is crucial. Montgomery (1994) considers a social norm, this paper provides more discussion on the decency norm in subsection 2.1.

that, at least in some cases, the poor opt for the latest option. This paper provides a formal model of endogenous time horizon that formalizes this change in attitude toward the future.

1.2 Anxiety and the anticipation of poverty

This paper argues that the anxiety from future poverty can be such that individuals may prefer being shortsighted in order to avoid the anxiety even though they are aware of the negative economic consequences of shortsightedness. Narayan and Ebrary (2000) asked 20,000 poor people across the developing world about their perception of poverty and found that anxiety represents a large part of the burden, in particular in Africa where, according to the authors, the sources of anxiety “are closely related to basic agriculture and survival that depend on the vagaries of nature, rains, droughts”. The book includes many quotes that describe the anxiety of the poor, among which the following ones:

Mental health problems—stress, anxiety, depression, lack of self-esteem and suicide—are among the more commonly identified effects of poverty and ill-being by discussion groups. In some African communities, people often describe a mental condition associated with poverty as “madness.”

As if land shortage is not bad enough we live a life of tension worrying about the rain: will it rain or not? There is nothing about which we say, “this is for tomorrow.” We live hour to hour. —A woman, Kajima, Ethiopia

“These agonizing decisions take their toll. People cope by focusing on one day at a time, becoming indifferent, apathetic or hovering near losing their mind.” A member of the research team in Ghana

In Malawi, ukavu means a state of constant deprivation. It is explained that households described in this group lack peace of mind because they are always worried about how to make ends meet. In most ukavu households, couples quarrel and fight a lot because they desire good lifestyles (umoyo uwemi), but they lack the means. “It is not surprising that most men from these households are drunkards because they drink to forget home problems.”

These quotes mark a distress caused by the inability to reach better economic prospects. The toll of anxiety is omnipresent, yet the most striking feature of these testimonies is the apparently irrational reaction to poverty by living day to day rather than making all possible efforts to plan a long-term exit strategy. This paper is, to the best of my knowledge, the first attempt to understand and evaluate the cause and consequences of the psychological mechanisms through which gloomy prospects discourage a poor individual from being forward looking.

The idea that future disutility is a source of distress is closely related to utility (or disutility) from anticipation, a concept that goes as far back as Bentham (1838). It was first applied in intertemporal choice by Jevons (1879) and his son Jevons (1905), and has been formalized by Loewenstein (1987)². While Jevons (1905) assumes that individuals always maximize their present utility, which incorporates utility from the anticipation of pleasure or pain, Loewenstein analyzes an agent that maximizes his intertemporal utility, given that the utility at each period incorporates the anticipation of future consumption. Loewenstein’s model explains why subjects are willing to pay a substantially higher amount to avoid an electric shock that is delayed than an immediate electric shock.

²For more historical background, Loewenstein and Elster (1992) provide an excellent review of the history of the economics of intertemporal choices.

With an exogenous preference for the present, the inclusion of anticipated utility makes individuals more willing to save. However, if one has the possibility to alter his focus toward the future, then he may close his eyes on the future to avoid the permanent distress of gloomy prospects. Whether the anticipation of future poverty can be such that individuals prefer a more acute, but myopic poverty is an empirical question that is investigated in section (3) of this paper.

Caplin and Leahy (2001) show how anticipatory utility can provide an explanation for many time inconsistencies by analyzing a model that incorporates the anxiety caused by uncertainty into the utility function. They claim that the cost of uncertainty about the future includes not only the loss caused by risk aversion, but also the psychological effect of this uncertainty. They provide an example of its impact on portfolio management, yet one can imagine that the anxiety is likely to be higher when facing a risk of starvation depending on the rainfall.

1.3 Getting into the brain

The recent progress in neurology brings many insights that help understand and model the human decision making process. McClure et al. (2004) recorded the neuro-images of subjects who were being asked to select from options that may or may not include immediate gratification. The researchers distinguish a “delta part” of the brain, which includes regions that are related to cognitive functions, from a “beta part” of the brain, which has consistently been implicated in impulsive behavior such as drug addiction. The delta part is activated similarly for all decisions, but the beta part is significantly more active in decisions that involve an immediate gratification. Individuals with more activation of the “delta part” are also more likely to opt for the delayed gratification. Besides this, Jamison and Wegener (2010) compare the neural activity during a decision between immediate and delayed gratification to the one during a decision between oneself and others and conclude that “the decision making process involving a tradeoff between our current and future selves is substantially the same as the decision making process involving a tradeoff between ourselves and other individuals”. Hence since Glass (1964) found that students asked to give electric shocks to victims increased their unfriendliness towards them, the same phenomena may occur between present and future selves. When the pressing needs of the present pushes the poor individual to “punish” future selves by saving very little money for them, relating less closely to the future selves allows the poor to reduce the cognitive dissonance and preserve their ego.

Together, these findings from neuro-images corroborate both economic models with a distinction between present and future selves (e.g Laibson (1997) and O’Donoghue and Rabin (1999)), but also models that incorporate two selves with different preferences at any single point in time. For example Thaler and Shefrin (1981) model a farsighted principal who controls the myopic agent through the alteration of his incentives and limitation of his opportunities. More recently Fudenberg and Levine (2006) show that this dual self model gives a unified explanation for several time inconsistencies. This paper also presents a dual self model that separates the economic decision from the endogenous determination of time discounting.

2 A model of cognitive dissonance and endogenous time horizon

This section presents a model where the cognitive dissonance appears in two different ways. First, a utility (or disutility) from the anticipation of consumption is incorporated. Hence when the future consumption is expected

to be below a decent living condition, then its anticipation generates the cognitive dissonance which reduces the utility of the individual. Second, the model has two selves. The “outer self” makes the consumption decision, taking his time preference as exogenous. The “inner self” determines the outer self’s time horizon. Although mostly subconscious, the determination of the time horizon is very rational since the inner self maximizes the intertemporal utility of the “whole self” without being biased towards any period. The time horizon is the extent to which the individual identifies with future selves at any given point in time. When a period is out of the individual’s time horizon, the individual does not identify with himself at that period, hence he does not plan and save for this future self nor does he feel any utility from anticipation from consumption at periods beyond his time horizon. A long term planning horizon requires the individual to face the potential distress caused by his gloomy economic prospects, which increases the cognitive dissonance. Consequently, even though the inner self is farsighted, she may opt for a low patience in order to reduce the permanent burden of the cognitive dissonance³. As shown in this section, the process of cognitive dissonance reduction has the side effect of reducing patience and can thus be the cause of a vicious cycle between poverty and myopia. This section first clarifies the introduction of the cognitive dissonance which is negative when consumption falls below a decency norm, and then provides an illustrative tail that provides an intuitive grasp of the theoretical model, before getting to the formal model and its conclusions.

2.1 Cognitive dissonance and the decency norm

The model incorporates a utility from anticipation, which, like Loewenstein (1987) is proportional to the actualized utility to be derived from future consumption. Third the cognitive dissonance is a source of disutility when a decent living condition will not be achieved in the future. Let z be the “decent” level of consumption, which can be interpreted as a local poverty line. Below z the lack of basic food, shelter, clothes and other needs creates anxiety as a result of anticipation. Above z , resources are used for other goods and services of which the anticipation a savoring⁴. Also let $\bar{u} = u(z)$ with $u(c)$ an increasing and concave utility function derived from immediate consumption. When $c = z$, the utility from anticipation should be null. The utility from anticipation of consumption at period t is proportional to $u(c_t) - \bar{u}$. Hence for a poor individual, the loss in utility from anticipation caused by the cognitive dissonance is proportional to the loss in utility caused by the poverty gap in period t . By normalizing $u(z) = 0$ ⁵ the utility from anticipation is proportional to $u(c_t)$, it is negative when the individual will be poor at time t and positive when he will be above the poverty line. Anchoring the utility function matters in models that give the possibility to experience or not a utility (e.g. Becker and Posner 2005), because adding a constant increases the willingness to experience this period.

2.2 An Illustrative Tale

This illustrative tale provides an intuitive grasp of the theoretical model. Miguel receives a meagre salary of \$300 on day one and needs to manage this amount over a period of 30 days, with no other source of income during the month. Miguel equally values every day of the month (no time discounting), has a decreasing marginal utility, and has no utility from anticipation. In a first scenario, Miguel has a planning horizon of 30 days (the entire duration of the game). Hence in the first period, he plans to consume \$10 per day, a plan that he manages to follow until the

³In order to distinguish them, I use the feminine for the inner-self and the masculine for the outer-self.

⁴Savoring commonly refers to a positive utility derived from the anticipation of consumption

⁵Hence for a log utility function $z = 1$

30th day. In a second scenario, Miguel is myopic and does not project himself farther than 10 days ahead, hence he always makes his consumption plan for the following 10 days. In the first period, Miguel thinks that he has \$300 to consume over 30 days and decides that he will consume \$10 per day, which he does in day one. However, when Miguel reaches day 2, he realizes that he has \$270 to be spent in the following 10 days (his horizon is still 10 days, but shifted forward by one day). Hence Miguel’s new plan is to consume \$27 per day from day 2 to day 12. Once again, in period 3, Miguel will need to adjust his consumption plan, because he is left with \$243 for 10 days of consumption, and will thus consume \$24.3 on day 3. Only when Miguel reaches period 21 will he be able to establish a consumption plan that he will follow until the end of the month, since he will not be “surprised” by the constant extension of the time horizon.

If Miguel could, at no cost, choose his planning horizon in a way that maximizes his actualized utility, he would opt for the 30 day time horizon given that it allows him to smooth his consumption much better than with a 10 day time horizon. Hence when the determination of the time horizon is free and costless, and in the absence of utility from anticipation, the individual will always opt for the highest possible time horizon.

Now let us introduce the utility from anticipation, and assume that it is negative for any day with a planned daily consumption that falls below the \$20 poverty line. In this case, though consumption choices would be different from the scenarios previously described, one can see how it raises a tradeoff in the determination of Miguel’s optimal time horizon. With a 30 days time horizon, Miguel will unavoidably face a distress that is caused by his inability to be above the poverty line during the 30 days. By contrast, with a 10 day period, the distress is reduced (at least during the initial phase), first because Miguel does not worry about any consumption beyond his 10 day horizon, and second because the illusion that he can consume all his wealth over 10 days makes him over-estimate his consumption during this period and thus reduces his anxiety related to the consumption within his time horizon. Hence for the poor, this creates a tradeoff between the reduction of psychological distress and a better consumption smoothing. Thus in this situation, the time horizon that maximizes Miguel’s intertemporal utility would be lower than 30 days. The following section formalizes this endogenous determination of the time horizon by an “inner self”, with a succession of consumption decisions at each period made by a potentially myopic “outer self”.

2.3 The decision process

The game describes an agent’s consumption decision from period 0 to T . It incorporates utility from anticipation and limited endogenous time horizon.

The succession of actions in this game is as follows:

Step 1: The inner self determines the horizon boundary H which is the maximum time horizon. The time horizon of each outer self i is constrained by the most binding element between the horizon boundary H and the end of the game T : $h_i = \min(H, T - i)$. When $h_i < T$, the outer self i is myopic, and the consumption plan is consistently updated.

Step 2: The outer self i (starting with $i = 0$) makes the consumption plan $\{c_t^i\}$ that maximizes his (myopic) intertemporal utility function, which incorporates the utility from consumption and its anticipation between period i and $i + h_i$. Only the immediate consumption c_i^i actually occurs in the consumption plan of the outer self i . The rest of the consumption plan of the outer self i , $\{c_t^i\}$ only affects the utility from anticipation at time i .

Step 3: The wealth increases (or decreases) by the difference between the returns from assets and the consumption c_t^i , and continuously, the following outer self repeats step 2) until $i = T$.

The following subsection provides a formal description of the three steps, starting with step 2 and 3 before introducing the inner self, because the inner self chooses H knowing the response function of the outer selves (the model is solved by backward induction).

2.3.1 The Succession of Outer Selves (step 2 and 3)

The outer self i acts as a typical consumer, maximizing his intertemporal utility function, although it has two specificities. First the utility from anticipation is incorporated and second, the outer self i is myopic when $h_i < T - i$. The outer self i , identifies with his future selves up to the time $i + h_i$ and considers the future selves after time $i + h_i$ to be another person that he is indifferent to⁶. This implies 1) that for the outer self i , only the consumption between i and h_i generates a utility from anticipation, 2) that the outer self i makes a consumption plan, given w_i and $\dot{w}_t = rw_t - c_t$ as if he will only live until time $i + h_i$ hence such that $w_{i+h_i} \geq 0$, and 3) that the consumption plan is consistently updated as time passes and the time horizon $[i, i + h_i]$ shifts forward.

For the outer self i with time horizon h_i , his utility at any given time t is composed of the utility from the instantaneous consumption c_t and anticipation of future consumption c_τ up to time $i + h_i$.

$$v_t^i(h_i, \{c_t\}) = e^{-\rho(t-i)}[u(c_t) + \gamma \int_t^{i+h_i} e^{-\rho(\tau-t)} u(c_\tau) d\tau] \quad (1)$$

Where v_t^i is the valuation for the outer self i of his utility at time t , which incorporates both $u(c_t)$ the utility derived from c_t the consumption at time t , and from the anticipation of consumption between time t and $i + h_i$ (after t , but within the time horizon of the outer self i). γ is an exogenous parameter that indicates the weight on utility from anticipation and ρ is the time discount factor⁷

The outer self i maximizes a utility function that incorporates the utility from anticipation:

$$\begin{aligned} \max_{\{c_t\}} U_i(\{c_t\}, h_i) &= \int_i^{i+h_i} v_t^i dt = \int_i^{i+h_i} e^{-\rho(t-i)} \left[u(c_t) + \gamma \int_t^{i+h_i} e^{-\rho(\tau-t)} u(c_\tau) d\tau \right] dt \\ &\quad s.t. \quad w_i = \bar{w}_i \\ &\quad \dot{w}_t = rw_t - c_t \\ &\quad w_h \geq 0 \end{aligned} \quad (2)$$

A continuum of outer selves i solve the maximization problem of equation 2 at time i . Let $\{c_t^i(h_i, w_i)\}$ be the consumption at time t planned by the outer self i that solves the intertemporal maximization problem of equation 2, and let $v_t^i(h_i, w_i) = v_t^i(h_i, \{c_t^i(h_i, w_i)\})$. Because the time horizon shifts forward with time, different outer selves (at different points in time i) can have different consumption plans given their horizon h_i and wealth w_i . Each outer

⁶A more progressive drop in the identification with future selves is discussed later

⁷Two forms of actualization appear in the utility from anticipation. $e^{-\rho(\tau-t)}$ indicates the extent to which anticipation of c_τ at time t decreases with the distance between t and τ , because a consumption that is farther away, generates a less intense utility from anticipation. $e^{-\rho(t-i)}$ is simply the actualization at time i of the utility from consumption or its anticipation at time t . Unlike Loewenstein (1987), the same time discount rate ρ applies to both forms of discount.

self i is only able to decide $\{c_t^i(h_i, w_i)\}$, the immediate consumption at time i , and the rest of his consumption plan ($\{c_t^i(h_i, w_i)\}$ for $i < t \leq i + h$) will generally not occur (unless the consumption plan is unchanged by the following outer selves), yet it matters because it determines the utility from anticipation of future consumption at time i , which enters in $v_i^i(h_i, w_i)$. Hence it is not the actual flow of consumption (the successive c_t^i for $0 \leq i \leq T$) that affects the utility from anticipation, but the anticipated consumption at each point i given time horizon h_i . Hence a poor agent has less anxiety from anticipation if he is myopic because the consumption beyond $i + h_i$ does not generate any utility from anticipation, but also because he naively believes that he will be able to consume all his wealth between i and $i + h_i$ and thus underestimate his poverty during this period.

At any given time i , the utility from consumption $c_i^i(h_i, w_i)$ and from the anticipation of the consumption plan $\{c_t^i(h_i, w_i)\}$ provide the instantaneous utility:

$$v_i^i(h_i, w_i) = u(c_i^i) + \gamma \int_i^{i+h_i} e^{-\rho(\tau-i)} u(c_\tau^i) d\tau \quad (3)$$

Continuously, the wealth changes following $\dot{w}_i = rw_i - c_i^i$ and the following agent repeats the same steps, solving again equation 2 from which the new consumption and instantaneous utility are obtained. The same process is repeated until $i = T$. Hence once all the outer selves have played, the agent (the whole self) receives the actualized utility:

$$U(H, w_0) = \int_0^T e^{-\rho(i)} v_i^i(h_i, w_i) di \quad (4)$$

After substituting equation 3:

$$U(H, w_0) = \int_0^T e^{-\rho(i)} \left[u(c_i^i) + \gamma \int_i^{i+h_i} e^{-\rho(\tau-i)} u(c_\tau^i) d\tau \right] di \quad (5)$$

with c_i^i and c_τ^i so that they solve the maximization problem of each outer self i , described in equation 2.

2.3.2 The Inner Self (step 1)

Aware of c_t^i , the response function of the outer selves and of $U(H, w_0)$, the inner self selects the horizon boundary $H \in (0, T]$, which maximizes the agent's intertemporal utility given by equation (5) :

$$\max_H U(H, w_0) = \int_0^T e^{-\rho(i)} \left[u(c_i^i) + \gamma \int_i^{i+h_i} e^{-\rho(\tau-i)} u(c_\tau^i) d\tau \right] di \quad (6)$$

such that $\{c_t^i\}$ solve the maximization problem of each outer self i , described in equation 2 and $h_i = \min(H, T - i)$. H represents the maximum time horizon of any outer self i , who has a time horizon equal to the most limiting

factor between the horizon boundary H and the end of the remaining time before the end of the game $T - i$. Let $H(w_0) \in (0, T]$ be the solution that maximizes equation 6. Notice that the inner self is far sighted in the sense that her objective function is the actualized utility of the whole self during the entire game.

2.4 Results

Propositions:

The propositions all assume a log utility function and their proofs are in the appendix.

Proposition 1: $\exists \hat{w}_0$ such that $H(w_0) < T \forall w_0 < \hat{w}_0$ and $H(w_0) = T \forall w_0 \geq \hat{w}_0$.

In words, there exists a level of wealth \hat{w}_0 such that when the initial wealth falls below \hat{w}_0 , then the inner self decides to be myopic in the sense that she selects a time horizon H lower than the duration of the game T .

Proposition 2: For any $w_0 < \hat{w}_0$ then $\frac{dH(w_0)}{dw_0} > 0$

This proposition states that, for any individual with a level of initial wealth below \hat{w}_0 , the time horizon of the individual is increasing in the initial wealth of the individual.

Proposition 3: If $w_0 < \hat{w}_0$ then $\frac{\partial \frac{w_i}{w_0}}{\partial w_0} > 0$ when $i < T - H(w_0)$

Proposition 3 highlights the divergence in the accumulation of assets. Individuals who are initially richer (starting with a higher w_0) will accumulate assets at a pace that is higher than individuals who are initially poorer. This is true as long as $i < T - H(w_0)$; since the duration of the game is finite and the model includes no bequest motive, all individual will start exhausting their wealth when reaching the end of the game in order to end with $w_T = 0$. Still individuals who are richer initially accumulate and thus generate more wealth during the course of their life.

A generalization of the horizon function

So far I have used a degenerate horizon function: $h(\theta, H) = \begin{cases} 1 & \text{if } \theta \leq H \\ 0 & \text{if } \theta > H \end{cases}$. Where $h(\theta, H)$ represents the

extent to which the outer-self at any time i identifies with himself at time $t = i + \theta$ The effective time discounting of the outer self at time i is thus given by $\int_i^T e^{-\rho\theta} h(\theta, H) c_t d_t$

Although the degenerate function offers an intuitive grasp of the time horizon, in practice it is likely that the time horizon of an individual drops more gradually. In the appendix, I generalize the same results to any function $h(\theta, H)$ that satisfies $\frac{dh(\theta, H)}{d\theta} \leq 0 \forall H \in (0, T]$ and $\frac{dh(\theta, H)}{dH} \geq 0 \forall \theta \in [i, T]$ and $\frac{dh(\theta, H)}{dH} > 0$ for at least some $\theta \in [i, T]$.

3 Empirical analysis

Context and data

This model predicts that among a relatively poor population, an increase in the initial or expected wealth of an individual should increase his patience. To test this prediction, I use the data from the project ‘‘Savings, subsidies,

and sustainable food security: A food experiment in Mozambique”. This project includes the distribution of randomly assigned vouchers that give rights to a 70% subsidy for a seed and fertilizer package for a half hectare of maize production. The distribution occurred in November-December 2010 and was followed by a survey of 1612 households in April-May 2011. The survey includes the question "How much time ahead do you plan your future expenditures?". I name the reply to this question the “horizon” variable. The question is inspired by Ameriks et al. (2003), who measured individuals’ propensity to plan and find that it has a strong impact on actual savings. Their questions, initially designed to capture the propensity to plan of highly educated Americans, was adapted to rural farmers in Mozambique. I use the horizon variable rather than typical time discounting questions (e.g “Do you prefer receiving x today or x a month from now?”) because the latter comprises not only time discounting but also the cost of remembering the debt and the trust that the money will be given a month later which may be accentuated by the low levels of income and education. Additionally, the answer is affected by the change in marginal utility between the time of the survey and one month after the survey. This is particularly problematic in the case of our project, given that farmers who received an agro-input subsidy do expect a larger harvest and thus a lower marginal utility about one month after the survey and would appear to be less patient. Hence, the replies to the typical time discounting questions would not only be noisy, but also biased against people who won the voucher lottery. Because the horizon variable is a new variable (to the extent of my knowledge), I first examine its correlation with other economic variables. Table 1a suggests that individuals who have a savings account, who have already received a formal credit, who used fertilizer for maize production during the previous campaign, or who are more optimistic about their future, all tend to have a higher time horizon than others. Although the causality is unknown, this suggests that the time horizon does have a relationship with key economic decisions.

The impact of receiving an agro-input subsidy on time horizon

The survey occurred before the harvest of 2011, corresponding to the campaign in which the fertilizer was subsidized for those who received a voucher. Hence the farmers who received a voucher experienced an exogenous increase in their economic prospect. The model thus predicts that these farmers should become more forward looking (as measured by the horizon variable).

The following regression provides an unbiased estimate of the intention to treat effect:

$$H_i = \beta_0 + \beta_1 T_i + \epsilon_i \tag{7}$$

where H_i is the horizon of individual i , and T_i indicates whether individual i has been randomly selected to be in the treatment group, i.e. to receive a voucher for agro-input subsidy. However, winning the lottery did not always translate to receiving the voucher because the result of the voucher lottery was not perfectly implemented and because farmers who were unable to complete the subsidy declined the voucher. An unbiased estimator of the impact of receiving the voucher on the time horizon can be obtained by an instrumental variable approach, where the first stage regressions is:

$$V_i = \alpha_0 + \alpha_1 T_i + v_i \tag{8}$$

where V_i indicates whether the individual i received a voucher according to his own reply to the survey.

$$H_i = \beta_0 + \beta_1 \hat{V}_i + \epsilon_i \quad (9)$$

with $\hat{V}_i = \alpha_0 + \alpha_1 T_i$

Table 3 shows the first stage of the IV regressions, which confirms that winning the lottery significantly increased the probability of receiving a voucher. Table 2 presents the regressions 7 and 9. I find that on average, winning the lottery increased an individual's time horizon by more than a month, from 198 days to 235 days, and increased it by about 13% on average. The effect is significant, and is only somewhat robust to the use of log of horizon instead of horizon. The third column indicates that once controlled for exogeneity bias, receiving a voucher increases the recipient's time horizon by more than three months. The results show that the time horizon of an individual is endogenous, and that it changed rapidly and substantially after receiving an agro-input subsidy.

Differential impact by income groups

The model predicts that the cognitive dissonance process affects relatively poor individuals, hence receiving the voucher should only increase the time horizon of individuals who were below a certain income threshold. The survey includes the maize production of the agricultural campaign preceding the treatment (2009-2010). Since maize is the major crop produced in the region, I use this data to separate the farmers into two groups, depending on whether they harvested more or less than two tons⁸ of maize in 2010. In Table 3, I find that receiving a voucher significantly increases the time horizon of the small maize producer by 113 days, while it increases it by 62 days for a large maize producer, and the latter effect is not significantly different from zero.

Falsification test

In a brief falsification test, I check whether winning the lottery has an impact on the farmers who did not receive their vouchers. I expect no impact of winning the lottery for this subgroup, given that all the benefits of winning the lottery are conditional on receiving the voucher. Table 5 confirms this expectation.

Impact of the Matched Savings intervention

A second intervention has been implemented starting in April 2011, following the baseline survey. One third of the sample was encouraged to open savings accounts through easier access and financial education, and another third was offered a matched savings on top of the same encouragement to save. The Matched Savings offers a bonus of 50% of the savings left in the account between harvest and the time to purchase fertilizer (from August 1st to October 31st). The financial intervention aims at helping farmers develop a habit of savings in order to carry forward the benefits of the agro-input subsidy from year to year. Table 6 compares the savings group to the matched savings group, allowing us to isolate the impact on the planning horizon of a subsidy that encourages savings. It excludes the control group since a financial training, which may have influenced the beneficiaries' time horizon, has been provided to both savings and matched savings groups but not to the control group. Table 6 shows that being selected to receive the matched savings increased the time horizon of the small maize producer by 29% on average, but it did not have any significant impact on the large maize producer, which provides further evidence that confirm the predictions from proposition one and two.

⁸While the threshold is arbitrary, generally a household that produces more than two tons of maize is above self sufficiency

Discussion of results

The impact of the medium-sized economic intervention is relatively high. The agro-input subsidy of a value equivalent to USD 65 increased the time horizon of small maize producers by more than a half, and the Matched Savings with an average transfer of USD 34 increased the time horizon of the small producers by 29% on average. These interventions were designed to leverage the amounts transferred, by requiring contributions from the farmers, and orienting it toward productive and forward looking activities. It is unlikely that a simple cash transfer of the same amount would have the same impact on the beneficiaries' planning horizon. Yet the results show that two different interventions which improve the economic prospects of the poor led to a substantial increase in his time horizon. Future research should address whether the change of attitude toward the future is permanent or temporary and how the planning horizon translates into economic decisions such as consumption, savings and investment. Although drawn from a very different context (a relatively highly educated population in the US), the results of Ameriks et al., 2003 indicate that planning is essential for savings and capital accumulation.

Are there alternative explanations of empirical results of this paper? This paper argues that the burden of the anticipation of poverty which makes poor individuals more prone to being shortsighted. Could there be a simple mechanical effect according to which having more money in hands leads to more planning? According to the model, having more money increases the benefits of planning ahead, because more money can be allocated more efficiently, but on the other side, being poor increases the necessity for planning because the marginal utility of consumption, and thus the marginal cost of a misallocation is higher. Intuitively, poor people need to accumulate assets and smooth their consumption as much as rich people do, hence it is not "rational" to be careless about one's long term needs even when the short term needs are not fully satisfied.

Banerjee and Mullainathan (2010) also predict a behavioral poverty trap, yet for very different reasons. They separate regular goods from temptation goods (for examples fatty or sugar ones) which are assumed to provide an immediate gratification, but are not valued by the long run self, and act as a temptation tax. They find that if this temptation tax rate is higher among the poor, then poor individuals may react to the prospect of future income growth by saving more. Yet the implications of their findings on an individual's time horizon are not explicit, hence it is not clear how it would provide an alternative explanation for the empirical results of this paper. However, it is not excluded that the two factors reinforce each other in the creation of a behavioral poverty trap.

4 Conclusion and policy implications

Effective development interventions require structural changes that will outlive the interventions. Poverty dynamics show that patience is a fundamental factor for an individual to be willing to make the investments and sacrifices required to transit toward a higher equilibrium. Changing financial practices is key to allowing the poor to make long-term plans and conceive exit strategies. This paper looks at the psychological causes behind myopic economic behaviors. It offers new ways to approach the issue, with empirical evidence of the endogeneity of the individual's planning horizon, and shows how it can result in a behavioral poverty trap that can occur even in the absence of non-convex production technologies. Using a field experiment among Mozambican maize producers, I find that benefitting from an agro-input subsidy or a savings subsidy increased significantly the planning time horizon of the small maize producers, but had no significant impact on large producers, as predicted by the theoretical model.

An economic intervention can affect the asset accumulation both through its direct economic impact, but also through a behavioral impact, which embraces all changes in preference, aspirations or attitudes that will in turn affect the economic decisions. Depending on the intervention, the behavioral impact may be positive (e.g. increase in patience or aspirations), or negative (e.g. increase in passivity or moral hazard). Improving the design of future projects requires a deep understanding of both the economic and behavioral impact of the interventions. While most of the literature has evaluated the economic impact, little is known about the behavioral effects. This paper shows that it is possible for typical impact evaluations to include an evaluation of changes in behavioral components that are expected to be affected by the treatments. It contributes to a better knowledge of the mechanisms that affect economic aspirations, which aims at encouraging the design of projects that enhance this effect. The type of intervention that deserve further attention include Individual Development Accounts, which offer a match at a fixed rate on savings towards the acquisition of assets. This approach stimulates long term investment both directly and indirectly because it generates “less fear and more hope” (Schreiner and Sherraden, 2007)

References

- Akerlof, G.A. and W.T. Dickens**, “The economic consequences of cognitive dissonance,” *The American Economic Review*, 1982, 72 (3), 307–319.
- Ameriks, J., A. Caplin, and J. Leahy**, “Wealth Accumulation and the Propensity to Plan*,” *Quarterly Journal of Economics*, 2003, 118 (3), 1007–1047.
- Appadurai, A.**, “The capacity to aspire: Culture and the terms of recognition,” *Culture and public action*, 2004, pp. 59–84.
- Ashraf, N., D.S. Karlan, and W. Yin**, “Tying Odysseus to the mast: Evidence from a commitment savings product in the Philippines,” *Yale University Economic Growth Center Discussion Paper No. 917*, 2005.
- Banerjee, A. and S. Mullainathan**, “The shape of temptation: Implications for the economic lives of the poor,” 2010.
- Becker, G.S. and C.B. Mulligan**, “The Endogenous Determination of Time Preference*,” *Quarterly Journal of Economics*, 1997, 112 (3), 729–758.
- and **R.A. Posner**, “Suicide and risk-taking: an economic approach,” *Unpublished Manuscript, The University of Chicago*, 2005.
- Bentham, J.**, “An introduction to the principles of morals and legislation (1789),” *The Works of Jeremy Bentham*, 1838, 43.
- Bernard, T., S. Dercon, and A.S. Taffesse**, “Beyond fatalism-an empirical exploration of self-efficacy and aspirations failure in Ethiopia.,” 2011.
- Bramel, D.**, “A dissonance theory approach to defensive projection,” *The Journal of Abnormal and Social Psychology*, 1962, 64 (2), 121.
- Brune, L., X. Giné, J. Goldberg, and D. Yang**, “Commitments to save: a field experiment in rural Malawi,” 2011.
- Caplin, A. and J. Leahy**, “Psychological Expected Utility Theory and Anticipatory Feelings*,” *Quarterly Journal of economics*, 2001, 116 (1), 55–79.
- Descartes, R.**, “1637,” *Discourse on Method*.
- Duflo, E. and A. Banerjee**, “The economic lives of the poor,” *Journal of Economic Perspectives*, 2007, 21 (1), 141–167.
- Festinger, L.**, *A theory of cognitive dissonance*, Stanford University Press, 1957.
- Fudenberg, D. and D.K. Levine**, “A dual-self model of impulse control,” *The American Economic Review*, 2006, 96 (5), 1449–1476.
- Glass, D.C.**, “Changes in liking as a means of reducing cognitive discrepancies between self-esteem and aggression1,” *Journal of Personality*, 1964, 32 (4), 531–549.

- Hirschman, A.O.**, “Obstacles to development: A classification and a quasi-vanishing act,” *Economic Development and cultural change*, 1965, 13 (4), 385–393.
- Jamison, J. and J. Wegener**, “Multiple selves in intertemporal choice,” *Journal of Economic Psychology*, 2010.
- Jevons, H.S.**, *Essays on economics*, Macmillan and co., limited, 1905.
- Jevons, W.S.**, *The theory of political economy*, Macmillan and co., 1879.
- Laibson, D.**, “Golden Eggs and Hyperbolic Discounting*,” *Quarterly Journal of Economics*, 1997, 112 (2), 443–477.
- Leibow, E.**, “Tally’s corner,” *Boston: Little, Brown*, 1967.
- Loewenstein, G.**, “Anticipation and the valuation of delayed consumption,” *The Economic Journal*, 1987, pp. 666–684.
- **and J. Elster**, *Choice over time*, Russell Sage Foundation Publications, 1992.
- , **E. Haisley, A. Caplin, and A. Schotter**, “The Economist as Therapist: Methodological ramifications of ‘light’ paternalism,” *Perspectives on the Future of Economics: Positive and Normative Foundations*, 2008, 1.
- Macours, K. and R. Vakis**, “Changing Households’ Investments and Aspirations through Social Interactions,” *Research Working papers*, 2009, 1 (1), 1–45.
- McClure, S.M., D.I. Laibson, G. Loewenstein, and J.D. Cohen**, “Separate neural systems value immediate and delayed monetary rewards,” *Science*, 2004, 306 (5695), 503.
- Montgomery, J.D.**, “Revisiting Tally’s Corner,” *Rationality and Society*, 1994, 6 (4), 462.
- Narayan, D. and Inc Ebrary**, *Voices of the Poor: Crying out for Change*, Oxford University Press for the World Bank New York and Oxford, 2000.
- O’Donoghue, T. and M. Rabin**, “Doing it now or later,” *American Economic Review*, 1999, 89 (1), 103–124.
- Pollak, R.A.**, “Consistent planning,” *The Review of Economic Studies*, 1968, 35 (2), 201–208.
- Rae, J.**, *Statement of some new principles on the subject of political economy: exposing the fallacies of the system of free trade, and of some other doctrines maintained in the " Wealth of nations. "*, Hillard, Gray, 1834.
- Ray, D.**, “Aspirations, poverty, and economic change,” *Understanding poverty*, 2006, pp. 409–21.
- Schreiner, M. and M. Sherraden**, *Can the poor save?: saving & asset building in individual development accounts*, Transaction Pub, 2007.
- Strotz, R.H.**, “Myopia and inconsistency in dynamic utility maximization,” *The Review of Economic Studies*, 1955, 23 (3), 165–180.
- Thaler, R.H. and H.M. Shefrin**, “An economic theory of self-control,” *The Journal of Political Economy*, 1981, 89 (2), 392–406.
- Trout, JD**, “THE PSYCHOLOGY OF DISCOUNTING: A POLICY OF BALANCING BIASES,” *Public affairs quarterly*, 2007, 21 (2), 201.

Table 1a: Basic Statistics

	Mean	SD	Min	10th pctile	Median	90th pctile	Max
Total land size (ha)	9.0	18.2	0	1.56	5	15	400
hh size	7.7	3.4	1	4	7	12	27
hh head educ (yrs)	4.6	3.2	0	0	4	9	13
urea (kg/ha)	6.2	20.7	0	0	0	20	300
npk (kg/ha)	5.5	18.2	0	0	0	16.7	185.2
maiz prod (kg)	2362	4669	0	360	1250	4980	126120
Yield (kg/ha)	847	849	0	180	583	1869	5934
Area maize (ha)	3.59	3.8	0.21	0.72	583	7	50
Horizon (days)	217.34	299.9	0	7	152.5	365	5840

Table 1b: Horizon mean and median conditional on economic variables:

		#	Horizon median	Horizon mean
Has a savings account	No	1237	152	210
	Yes	304	244	262
Used Fertilizer in 2009-2010 campaign	No	1251	122	215
	Yes	284	183	237
Cultivated more than 2ha of maize in 2009-2010 campaign	No	709	122	206
	Yes	814	183	230
Has already received a credit from a formal bank	No	1355	91	207
	Yes	188	152	211
Believes that the hh's economic situation will be better in 5 years	No	163	61	160
	Yes	1368	183	226

Table 2: Impact of the voucher subsidy program on time horizon

	ITT - OLS		ATE - IV	
	Horizon	Loghorizon	Horizon	Loghorizon
wonvoucherlot	37.46** [0.019]	0.13 [0.143]		
receivedvoucher			97.51** [0.020]	0.34 [0.141]
Constant	198.51*** [0.000]	4.36*** [0.000]	365.00*** [0.000]	5.90*** [0.000]
Observations	1,546	1,546	1,546	1,546
R-squared	0.085	0.077	0.072	0.086

Regressions include village fixed effects

Robust pval in brackets, *** p<0.01, ** p<0.05, * p<0.1

Table 3: Impact of the voucher subsidy program on time horizon by income groups

	Small maize producer (< 2 tons)				Large maize producer (>= 2 tons)			
	ITT - OLS		ATE - IV		ITT - OLS		ATE - IV	
	Horizon	Loghorizon	Horizon	Loghorizon	Horizon	Loghorizon	Horizon	Loghorizon
wonvoucherlot	46.25*** [0.005]	0.22** [0.047]			22.58 [0.523]	0.07 [0.697]		
receivedvoucher			113.95*** [0.007]	0.54** [0.046]			62.59 [0.522]	0.18 [0.697]
Constant	174.76*** [0.000]	4.23*** [0.000]	-220.91*** [0.009]	1.00* [0.066]	247.02*** [0.000]	4.58*** [0.000]	-55.59 [0.569]	1.90*** [0.000]
Observations	993	993	993	993	503	503	503	503
R-squared	0.080	0.090	0.037	0.097	0.189	0.206	0.197	0.212

Regressions include village fixed effects

Robust pval in brackets, *** p<0.01, ** p<0.05, * p<0.1

Table 4: First stage for IV regressions (receiving a voucher is the explained variable)

	All	Small maize producers	Large maize producers
wonvoucherlot	0.38*** [0.000]	0.41*** [0.000]	0.36*** [0.000]
Constant	0.12*** [0.000]	0.08*** [0.000]	0.20*** [0.000]
Observations	1,546	993	503
R-squared	0.215	0.227	0.300

Regressions include village fixed effects

Robust pval in brackets, *** p<0.01, ** p<0.05, * p<0.1

Table 5: falsification test: impact of winning lottery when not receiving voucher

	ITT - OLS	
	Horizon	Loghorizon
wonvoucherlot	7.25 [0.719]	-0.12 [0.314]
Constant	201.18*** [0.000]	4.35*** [0.000]
Observations	1,076	1,076
R-squared	0.088	0.111

Dependent variable is the horizon in number of days

pval in brackets, *** p<0.01, ** p<0.05, * p<0.1

Includes only individuals who did not receive a voucher

Table 6: Impact of the matched savings on time horizon (compared to treatment group with same encouragement to save but no matched savings)

	All		Small maize producers		Large maize producers	
	Horizon	Loghorizon	Horizon	Loghorizon	Horizon	Loghorizon
Matched savings	22.27 [0.243]	0.12 [0.179]	55.47* [0.093]	0.29** [0.012]	-27.61 [0.253]	-0.07 [0.649]
Constant	161.39*** [0.000]	4.29*** [0.000]	137.90*** [0.000]	4.17*** [0.000]	199.83*** [0.000]	4.45*** [0.000]
Observations	961	961	605	605	320	320
R-squared	0.045	0.047	0.079	0.064	0.165	0.168

Standard errors are clustered at the locality level (level of randomization)

Regressions include fixed effects at the stratification (groups of 3 localities by proximity)

pval in brackets, *** p<0.01, ** p<0.05, * p<0.1

Appendix

Assuming a log utility function, I solve the model, before providing a proof of the propositions.

The outer self's maximization

I solve the outer self's problem to obtain $\{c_t^i(h_i, w_i)\}$:

The following equation is equivalent to equation 2 (under the same constraints):

$$\max_{\{c_t\}} U_i(\{c_t\}, h_i) = \int_i^{i+h_i} \underbrace{e^{-\rho(t-i)} u(c_t)}_{\text{Utility from consumption}} + \underbrace{\gamma \int_i^t e^{-\rho(\tau-i)} e^{-\rho(t-\tau)} d\tau u(c_t)}_{\text{Utility from anticipation}} dt \quad (10)$$

Equation 2 is the sum of the utility (from instantaneous consumption and anticipation) at every point in time between i and h_i while equation 10 is the sum of the utility (from instantaneous consumption and anticipation) derived from every consumption c_t between i and h_i , which is equivalent.

Since $\gamma \int_i^t e^{-\rho(t-i)} u(c_t) dt = \gamma(t-i) e^{-\rho(t-i)} u(c_t)$; I simplify equation 10 to obtain:

$$\max_{\{c_t\}} U_i(\{c_t\}, h_i) = \int_i^{i+h_i} e^{-\rho(t-i)} (1 + \gamma(t-i)) u(c_t) dt \quad (11)$$

The marginal utility of increasing consumption at any time t , is given by the derivative with respect to c_t : $e^{-\rho(t-i)} (1 + \gamma(t-i)) u'(c_t)$

Given the interest rate r , the cost of consuming one unit at time t is: e^{rt}

At the optimum the marginal utility derived from one monetary unit dedicated to consumption must be the same for every period:

$$\frac{e^{-\rho(s-i)} (1 + \gamma(s-i)) u'(c_s)}{e^{r(s-i)}} = \frac{e^{-\rho(t-i)} (1 + \gamma(t-i)) u'(c_t)}{e^{r(t-i)}} \quad \forall s, t \in [i, i + h_i] \quad (12)$$

I use a log utility function and simplify to obtain:

$$\frac{e^{(r-\rho)(s-i)} (1 + \gamma(s-i))}{c_s} = \frac{e^{(r-\rho)(t-i)} (1 + \gamma(t-i))}{c_t} \quad \forall s, t \in [i, i + h_i] \quad (13)$$

I introduce the budget constraint: $\int_i^{i+h_i} e^{r(t-i)} c_t dt = w_i$, and obtain:

$$c_t^i(h_i, w_i) = w_i \frac{e^{(r-\rho)(t-i)} (1 + \gamma(t-i))}{\int_i^{i+h_i} e^{-\rho(s-i)} (1 + \gamma(s-i)) ds} \quad (14)$$

Hence the utility from anticipation at time i from horizon h_i and consumption plan $\{c_t^i\}$ is given by:

$$UA_i(\{c_t^i\}, h_i) = \gamma \frac{\int_i^{i+h_i} e^{-\rho(t-i)} (t-i) u(c_t^i(h_i, w_i)) dt}{\int_i^{i+h_i} e^{-\rho(s-i)} (1 + \gamma(s-i)) ds} \quad (15)$$

With $\{c_t^i(h_i, w_i)\}$ defined in equation 14

When $i \leq T - H$, the immediate consumption that will actually happen at time i is thus:

$$c_t^i(h_i, w_i) = \frac{w_i}{\int_i^{i+h_i} e^{-\rho(s-i)} (1 + \gamma(s-i)) ds} \quad \forall i \leq T - H \quad (16)$$

From $\dot{w}_t = rw_t - c_t$, we obtain :

$$\frac{\dot{c}_t^i}{c_t^i} = \frac{\dot{w}_t}{w_t} = r - \frac{1}{\int_i^{i+h_i} e^{-\rho(s-i)} (1 + \gamma(s-i)) ds} \quad \forall i \leq T - H \quad (17)$$

After solving for the integral and defining w_i with relation to w_0 I obtain:

$$w_i(w_0, H) = w_0 e^{i(r - \frac{\rho^2}{\gamma + \rho - e^{-H\rho(\gamma + \rho + H\rho)}})} \quad \forall i \leq T - H \quad (18)$$

I substitute equation 18 into equations 14 and 16 to obtain the following ones:

$$c_t^i(h_i, w_i) = w_0 e^{i(r - \frac{\rho^2}{\gamma + \rho - e^{-H\rho(\gamma + \rho + H\rho)}})} \frac{e^{(r-\rho)(t-i)} (1 + \gamma(t-i))}{\int_i^{i+h_i} e^{-\rho(s-i)} (1 + \gamma(s-i)) ds} \quad \forall 0 \leq i \leq t \leq T - H \quad (19)$$

When $H \geq T - i$, $c_t^i(h_t, w_t) = c_t^i(h_i, w_i)$ for $t > i$ which means that the consumption plan of the outer self are consistent and defined by equation 14 once $i = T - H$ is reached.

$$c_t^i(h_i, w_i) = w_{T-H} \frac{e^{(r-\rho)(t-T+H)} (1 + \gamma(t-T+H))}{\int_i^{i+h_i} e^{-\rho(s-T+H)} (1 + \gamma(s-T+H)) ds} \quad \forall T - H \leq i \leq t \leq T \quad (20)$$

After plugging $w_i(w_0, H)$ from equation 18:

$$c_t^i(h_i, w_i) = w_0 e^{(T-H)(r - \frac{\rho^2}{\gamma + \rho - e^{-H\rho(\gamma + \rho + H\rho)}})} \frac{e^{(r-\rho)(t-T+H)} (1 + \gamma(t-T+H))}{\int_i^{i+h_i} e^{-\rho(s-T+H)} (1 + \gamma(s-T+H)) ds} \quad \forall T - H \leq i \leq t \leq T \quad (21)$$

The inner self's maximization

I take the derivative of $U(H, w_0)$ with respect to H to obtain the first order condition:

$$\frac{dU(H, w_0)}{dH} = \int_0^T e^{-\rho i} u'(c_i^i) \frac{dc_i^i}{dH} + \gamma \int_i^{i+h_i} e^{-\rho \tau} u'(c_\tau^i) \frac{dc_\tau^i}{dH} d\tau di + \gamma \int_0^{T-H} e^{-\rho H} u(c_{i+H}^i) di = 0 \quad (22)$$

Equations 19 and 21 show that c_t^i is proportional to $w_0 \forall 0 \leq i \leq t \leq T$, which is not surprising given the use of a log utility function. It implies that $u'(c_t^i) \frac{dc_t^i}{dH}$ is independent from w_0 . Hence in equation 22, only the third element is a function of w_0 , which I derive to find that:

$$\frac{d^2U(H(w_0), w_0)}{dH \partial w_0} = \gamma \int_0^{T-H} e^{-\rho H} u'(c_{i+H}^i) \frac{\partial c_{i+H}^i}{dw_0} di > 0 \quad (23)$$

Proof of propositions 1 and 2:

The solution $H(w_0)$ to the inner self's problem is either an interior solution, satisfying $H(w_0) < T$ and the first and second order conditions $\frac{dU(H, w_0)}{dH} = 0$ and $\frac{d^2U(H, w_0)}{dH^2} < 0$; or it is a corner solution satisfying $H(w_0) = T$ and $\frac{dU(T, w_0)}{dH} > 0$ ⁹.

In the case of an interior solution, an implicit differentiation of $\frac{dU(H, w_0)}{dH} = 0$ give us:

$$\frac{d^2U(H, w_0)}{dH^2} \frac{dH(w_0)}{dw_0} + \frac{\partial \frac{dU(H, w_0)}{dH}}{\partial w_0} = 0 \quad (24)$$

Hence:

$$\frac{dH(w_0)}{dw_0} = - \frac{\frac{d^2U(H(w_0), w_0)}{dH \partial w_0}}{\frac{d^2U(H, w_0)}{dH^2}} > 0 \quad (25)$$

Since $\frac{d^2U(H, w_0)}{dH \partial w_0} > 0$, \exists a unique \hat{w}_0 such that $\frac{dU(T, \hat{w}_0)}{dH} = 0$. When $w_0 > \hat{w}_0$ then $\frac{dU(T, \hat{w}_0)}{dH} > 0$ and $H(w_0) = T$ is the solution. When $\frac{dU(T, \hat{w}_0)}{dH} < 0$, then since

From equation 23, we know that $\frac{dU(H=T, w_0)}{dH}$ is strictly increasing in w_0 . Hence there exists only one \hat{w}_0 such that $\frac{dU(H=T, w_0)}{dH} = 0$. When $w_0 > \hat{w}_0$ then $\frac{dU(T, \hat{w}_0)}{dH} > 0$ and $H(w_0) = T$ is the solution. When $w_0 < \hat{w}_0$, then equation 25 tells us that $H(w_0) < H(\hat{w}_0) = T$ and that $H(w_0)$ is an increasing function. I have proved proposition 1 and 2.

Proof of propositions 3:

According to equation 17 then: $\frac{\dot{w}_i}{w_i} = r - \frac{1}{\int_i^{i+h_i} e^{-\rho(s-i)} (1+\gamma(s-i)) ds} \forall i \leq T - H$

When $i \leq T - H$, then $\int_i^{i+h_i} e^{-\rho(s-i)} (1+\gamma(s-i)) ds$ is an increasing function of H (since $h_i = \min(H, T - i)$, hence $\frac{\dot{w}_i}{w_i}$ is also strictly increasing in H .

Proof of propositions with a general smooth horizon function:

The maximization problem of the inner self becomes the following:

⁹These are sufficient conditions for a local maximum and necessary conditions for a global maximum

$$\max_H U(H, w_0) = \int_0^T e^{-\rho(i)} \left[u(c_i^i) + \gamma \int_i^T h(\tau - i, H) e^{-\rho(\tau-i)} u(c_\tau^i) d\tau \right] \quad (26)$$

The new first order condition with respect to H is as follows:

$$\frac{dU(H, w_0)}{dH} = \int_0^T e^{-\rho i} u'(c_i^i) \frac{dc_i^i}{dH} + \gamma \int_i^T h(\tau - i, H) e^{-\rho \tau} u'(c_\tau^i) \frac{dc_\tau^i}{dH} d\tau di + \gamma \int_0^T \int_i^T e^{-\rho(\tau-i)} h(\tau - i, H) u(c_\tau^i) d\tau di = 0 \quad (27)$$

For the same reason as in the previous case, c_τ^i and c_i^i are proportional to income and thus only the third element of equation 27 remains to be derived to obtain the derivative of the first order condition with respect to w_0 .

$$\frac{d^2 U(H(w_0), w_0)}{dH \partial w_0} = \gamma \int_0^T \int_i^T \frac{dh(\tau - i, H)}{dH} e^{-\rho(\tau-i)} h(\tau - i, H) u(c_\tau^i) d\tau di > 0 \quad (28)$$

The remaining part of the proof of the propositions is unchanged.